A Hybrid Iterative Decomposition Approach to ROADEF/EURO 2010

D. Anghinolfi, C. Nattero, M. Paolucci, [1]
L. M. Gambardella, R. Montemanni, N. E. Toklu [2]

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[1] Universitá degli Studi di Genova,
[2] Istituto Dalle Molle per gli Studi sull'Intelligenza Artificiale

Part I

Problem Description

minimize energy production costs
of a (large) set of power plants
while satisfying customer demand

- schedule outages and plan reload for nuclear plants
- demand is uncertain and estimated through scenarios
- choose production levels in each scenario
- large scale: many plants, many scenarios, long time horizon

Technical Constraints

1. bounds on power output [CT2-5]
2. bounds on modulation [CT12]
3. power profile when fuel stock is under a threshold [CT6]
4. bounds on refuelling [CT7]
5. bounds on fuel stock [CT8-11]

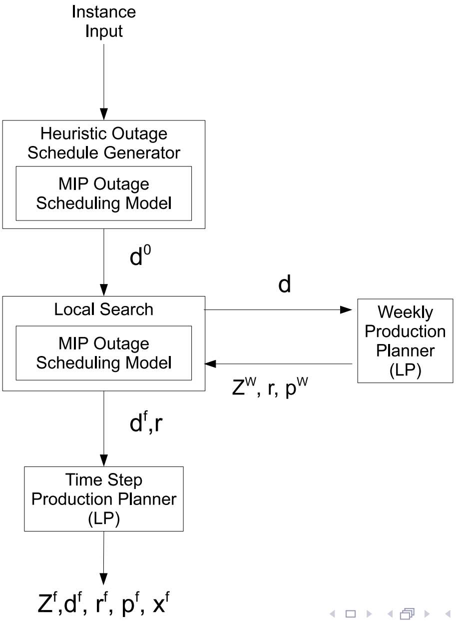
Scheduling Constraints

1. given interval for each outage [CT13 - CT13 bis]
2. min spacing / max overlapping between outages [CT14 - 15]
3. min spacing between decoupling dates [CT16]
4. min spacing between dates of coupling [CT17]
5. min spacing between coupling and decoupling dates [CT18]
6. use of resources [CT19]
7. max number of overlapping outages [CT20]
8. offline power capacity [CT21]

Part II

The approach

An Overview of the Solution Approach



A Comparison of the models

Module	HOSG and LS	WPP	TSPP
Model Type	MIP	LP	LP
Objective	N/A	weekly	t.1 + deviation
Resolution	week	week	time step
Scenarios	N/A	all	one at a time
Notes	feasibility	"macro" t.1	need post proc.

Heuristic Outage Schedule Generator (HOSG)

Defines a MIP at each iteration:

- Detailed scheduling constraints
- rough approximation of stocks and refuellings

Constraint satisfaction problem (no optimization)
Idea: more outages $\Rightarrow$ more type 2 power $\Rightarrow$ less type 1 costs

Procedure outline:

1. schedule all mandatory outages
2. parallel phase: add optional cycles to each plant, beginning from the smallest. Iterate if successful
3. stochastic phase: choose a small subset of plants and increment optional cycles

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MIP Outage scheduling model

Sets and Indices

- $i \in \mathbb{I} \triangleq \{0, \ldots, I-1\}$ is the index of type 2 power plants
- $k \in \mathbb{K} \triangleq \{-1, 0, \ldots, K-1\}$ is the index of cycles
- $h \in \mathbb{W} \triangleq \{0, \ldots, H\}$ is the index of weeks

Decision Variables

- $d_{i,k,h} \in \mathbb{B} = 1$ if plant i begins the k-th outage in week h. $\forall i, k, h : N^L_{ik} \leq h \leq N^U_{ik}$. It is the *decoupling date*
- $x^B_{ik} \in \mathbb{R}_+$ is the remaining stock of fuel in plant i at the end of cycle $k-1$, just *before* cycle k
- $x^A_{ik} \in \mathbb{R}_+$ is the initial stock of fuel in plant i at the beginning of cycle k, right *after* cycle $k-1$
- $r_{ik} \in \mathbb{R}_+$ is the *reload*

Cumulative Maximum Consumption up to week h

$$\mathrm{P}_{ih}^{\mathrm{max}} \triangleq \sum_{z=0}^{h} PMAX_{iz} \cdot D_{z}^{\mathbb{H}}$$

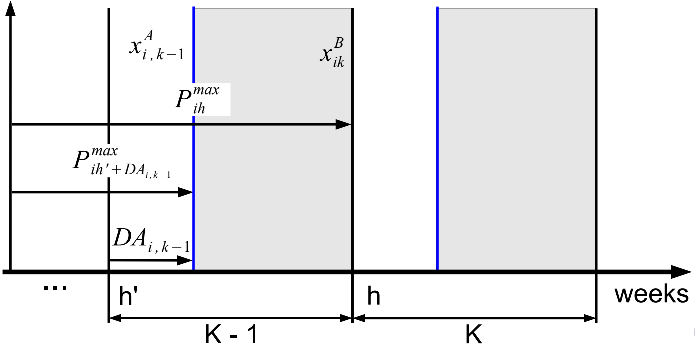
Cumulative Maximum Consumption in cycle $k-1$

$$\sum_{h=0}^{H-1} d_{ikh}\mathrm{P}_{ih}^{\mathrm{max}} - \sum_{h=0}^{H-1-DA_{ik}} d_{i,k-1,h}\mathrm{P}_{i,h+DA_{i,k-1}}^{\mathrm{max}}$$

[CT9] (and partly [CT13])

$$x_{ik}^{B} \geq x_{i,k-1}^{A} - \sum_{h=0}^{H-1} d_{ikh} \mathrm{P}_{ih}^{\max} + \sum_{h=0}^{H-1-DA_{ik}} d_{i,k-1,h} \mathrm{P}_{i,h+DA_{i,k-1}}^{\max} - d_{ikH} M + d_{i,k-1,H} M$$

$\forall i, k$



... the other conditions on stock and refuelling

$$R_{ik}^{\min} \cdot (1 - d_{ikH}) \leq r_{ik} \leq R_{ik}^{\max} \cdot (1 - d_{ikH}) \quad \forall i, k \quad \text{[CT7]}$$

$$x_{ik}^{A} = R_{ik}^{r} \cdot (x_{ik}^{B} - BO_{i,k-1}) + r_{ik} + BO_{i,k-1} \quad \forall i, k \quad \text{[CT10]}$$

$$x_{ik}^{B} \leq AMAX_{i,k} \quad \forall i, k \quad \text{[CT11]}$$

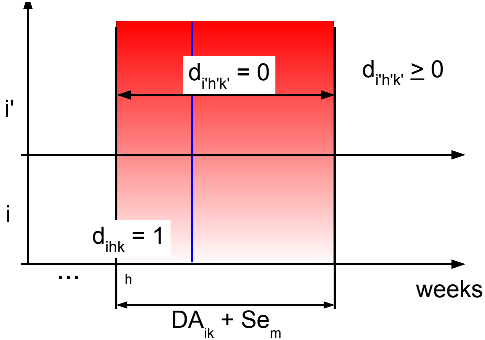
$$x_{ik}^{A} \leq SMAX_{i,k} \quad \forall i, k \quad \text{[CT11]}$$

$$x_{i,-1}^{A} = XI_{i} \quad \forall i \quad \text{[CT8]}$$

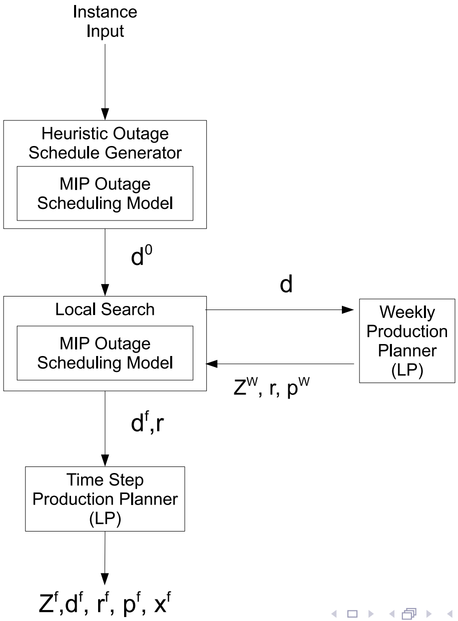
an example for [CT14]

$$d_{ikh} + \sum_{h'=h}^{\min(H-1,h+DA_{ik}+Se_f)} d_{i'k'h'} \leq 1$$

$$\forall f, i, i', k, h : f \in \mathrm{CT}_{14} \ \wedge i, i' \in A_f \ \wedge i \neq i'$$



Next step: Local Search (LS)



trajectory based, starts search from given initial solution

Outline

1. greedy randomized behaviour
2. switches to Simulated Annealing when gets stuck
3. black list for unfeasible solutions
4. randomized restarts

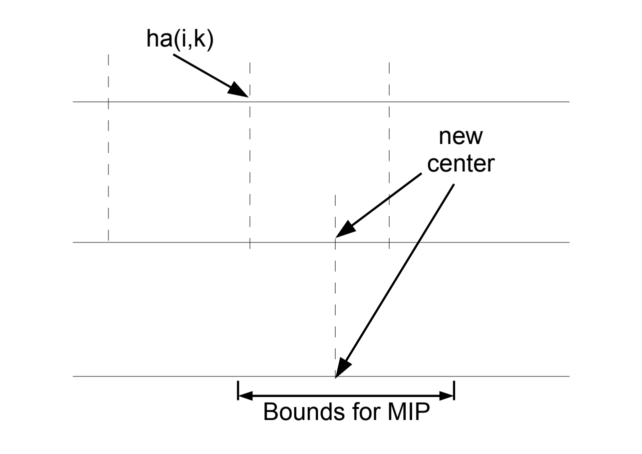
A Solution Vector

act(i)	k=0	k=1	k=2	...	k=K-1
act(0)	ha(0,0)	ha(0,1)	ha(0,2)	...	ha(0,K-1)
act(1)	ha(1,0)	ha(1,1)	ha(1,2)	...	ha(1,K-1)
act(2)	ha(2,0)	ha(2,1)	ha(2,2)	...	ha(2,K-1)
...	...	...	...	...	...
act(I-1)	ha(I-1,0)	ha(I-1,1)	ha(I-1,2)	...	ha(I-1,K-1)

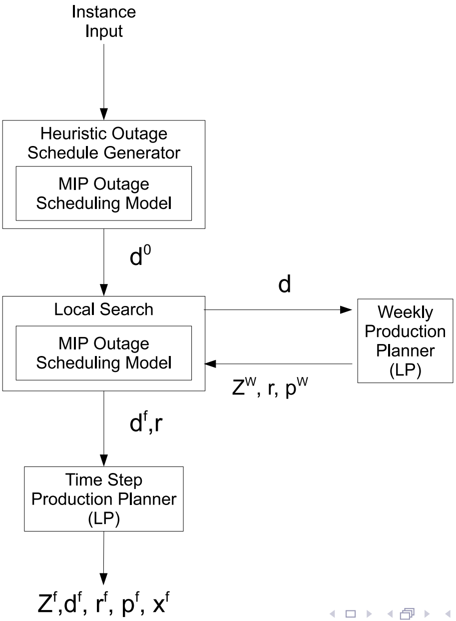
where,

act(i) = "Cycle activation" value of power plant i
ha(i,k) = Decoupling week of cycle k of power plant i
K = Number of cycles
I = Number of type 2 power plants

Moves



Evaluation Module: Weekly Production Planner (WPP)



a LP for the Weekly Production Planner

Sets and Indices

- $s \in \mathbb{S} \triangleq \{0, \ldots, S-1\}$ is the index of scenarios
- $i \in \mathbb{I} \triangleq \{0, \ldots, I-1\}$ is the index of type 2 power plants
- $k \in \mathbb{K} \triangleq \{-1, 0, \ldots, K-1\}$ is the index of cycles
- $t \in \mathbb{T} \triangleq \{0, \ldots, T-1\}$ is the index of steps
- $h \in \mathbb{W} \triangleq \{0, \ldots, H\}$ is the index of weeks
- *(*$j \in \mathbb{J} \triangleq \{0, \ldots, J-1\}$ *is the index of type 1 power plants)*

Decision Variables

- p^1_{hs} is the power output of "macro" type 1 plant
- p^2_{ihs} is the power output of type 2 power plant i
- $x^B_{ik} \in \mathbb{R}_+$ is the remaining stock of fuel in plant i at the end of cycle $k-1$
- $x^A_{ik} \in \mathbb{R}_+$ is the initial stock of fuel in plant i at the beginning of cycle k
- $r_{ik} \in \mathbb{R}_+$ is the refuelling of plant i in cycle k
- x^f_{is} is the final fuel stock

construction of "macro" type 1 plant

1. sort all the plants by production cost (increasing order)
2. satisfy demand using only type 1 plants. Use the full production capacity of a plant before using the next one (begin using the cheapest one)
3. the equivalent cost of production C^1_{hs} is calculated according to the following definition:

$$C^1_{hs} \triangleq \frac{\sum_{j \in \mathbb{J}} C_{jhs} \cdot \tilde{p}_{jhs}}{L^{\mathbb{H}}_h}$$

Objective Function

$$\min \ \frac{1}{|S|} \sum_{s \in \mathbb{S}} \left[\sum_{h \in \mathbb{H}} \left(p^1_{hs} \cdot L^{\mathbb{H}}_h \cdot C^1_{hs} \right) - \sum_{i \in \mathbb{I}} C_{iH} \cdot x^f_{is} \right] + \sum_{i \in \mathbb{I}} \sum_{k \in \mathbb{K}} C_{ik} \cdot r_{ik}$$

Fuel Stock Variation during Production [CT9]

$$x_{iks}^{B} = x_{i,k-1,s}^{A} - \sum_{h=ha(i,k-1)+DA_{i,k-1}}^{ha(i,k)} p_{ihs}^{2} \cdot L_h^{\mathbb{H}} \quad \forall i, k, s$$

. . . the other conditions on Stock and Refuelling

$$x^A_{iks} = R^r_{ik} \cdot \left(x^B_{iks} - BO_{i,k-1}\right) + r_{ik} + BO_{ik} \quad \forall i, k, s$$
$$x^B_{iks} \leq AMAX_{ik} \quad \forall i, k, s$$
$$x^A_{iks} \leq SMAX_{ik} \quad \forall i, k, s$$

Maximum Modulation [CT12]

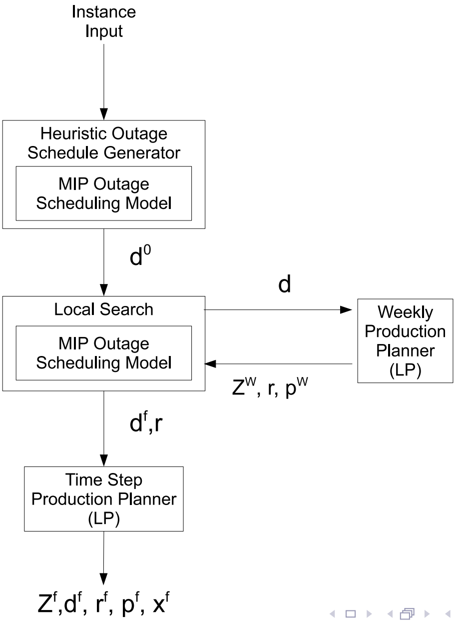
$$\sum_{h=ha(i,k)}^{N_{ik}^{\min}} \left[PMAX_{ih} - p_{ihs}^{2} \cdot L_{h}^{\mathbb{H}} \right] \leq MMAX_{ik} \quad \forall i, k, s$$

$$p_{ihs}^{2} = 0 \quad \forall i, k, h, s : N_{ik}^{\max} \leq h \leq ha(i,k) + DA_{ik}$$

Modulation Reference

$$\text{mod}_{iks} = \min \left[\left(\sum_{h \in [ha(i,k), ha(i,k)+DA_{ik}]} P_{ih}^{\max} - p_{ikh}^2 \cdot L_h^{\mathbb{H}} \right) \ , \ \ MMAX_{i,k} \right]$$

Final module: Time Step Production Planner (TSPP)



Overview of the Time Step Production Planner (TSPP)

for all scenarios:

1. check and repair fuel stock
2. - populate and solve LP model
   - deviational variables Δ allow modulation correction. This phase is iterated until convergence.
3. repeat until fuel corrections are no longer necessary

A LP for the Time Step Production Planner (TSPP)

Sets and Indices

- $j \in \mathbb{J} \triangleq \{0, \ldots, J-1\}$ is the index of type 1 power plants
- $i \in \mathbb{I} \triangleq \{0, \ldots, I-1\}$ is the index of type 2 power plants
- $k \in \mathbb{K} \triangleq \{-1, 0, \ldots, K-1\}$ is the index of cycles
- $t \in \mathbb{T} \triangleq \{0, \ldots, T-1\}$ is the index of steps
- $h \in \mathbb{W} \triangleq \{0, \ldots, H\}$ is the index of weeks
- ($s \in \mathbb{S} \triangleq \{0, \ldots, S-1\}$ *is the index of scenarios*)

Note: one scenario at a time!

Decision Variables

- $p^{1s}_{jt} \in \mathbb{R}_+$, the production of type 1 power plant j in time step t in scenario s
- $p^{2s}_{it} \in \mathbb{R}_+$, the production of type 2 power plant i in time step t in scenario s
- $\Delta^+_{ik} \in \mathbb{R}_+$, the deviational variable for positive violation of the reference modulation for plant i in scenario k
- $\Delta^-_{ik} \in \mathbb{R}_+$, the deviational variable for negative violation of the reference modulation for plant i in scenario k

A LP for the Time Step Production Planner (TSPP)

Objective Function

$$\min \frac{1}{S} \sum_{j} \sum_{t} \left[C_{jts} \cdot L_t^{\mathbb{T}} \cdot p_{jt}^{1s} \right] + M \cdot \sum_{i} \sum_{k} \left[\Delta_{ik}^{+} + \Delta_{ik}^{-} \right]$$

A LP for the Time Step Production Planner (TSPP)

Modulation Correction

$$\sum_{t \in \mathbb{T}^P_{ik}} p^{2s}_{it} \cdot L^{\mathbb{T}}_t + \Delta^+_{ik} - \Delta^-_{ik} = \sum_{t \in \mathbb{T}^P_{ik}} P^{\max}_{ik} \cdot L^{\mathbb{T}}_t - \mathrm{mod}^{\max}_{ik} \quad \forall i, k$$

$$\Delta^+ \leq \max\left(MMAX_{ik} - \mathrm{mod}^{\max}_{ik}, 0\right) \quad \forall i, k$$

$$\Delta^- \leq MMAX_{ik} \quad \forall i, k$$

A LP for the Time Step Production Planner (TSPP)

Power Profile [CT6]

$$p_{it}^{2s} = \mathrm{prof}\left(x_{its}\right) \cdot P_{ik}^{\max} \quad \forall i, k, t : t \in \mathbb{T}_{ik}^{P} \wedge x_{it} < BO_{ik}$$

Fuel Stock Check and Correction

- idea: calculate the fuel stock given the initial value and the power output in each time step
- increment or reduce the refuelling where necessary, until constraints are respected
- no stock corrections? Then the solution is coherent. TSPP can pass on to next scenario

Part III

Results and conclusions

on dataset B

data B6:	91·462·048·423.964417	$\simeq 91.462 \cdot 10^9$
data B7:	86·688·984·427.134369	$\simeq 86.688 \cdot 10^9$
data B8:	415·438·385·373.674010	$\simeq 415.438 \cdot 10^9$
data B9:	318·260·189·684.773130	$\simeq 318.260 \cdot 10^9$
data B10:	85·487·064·852.524277	$\simeq 85.487 \cdot 10^9$

that's it

Thank you for your attention :-)

on dataset B

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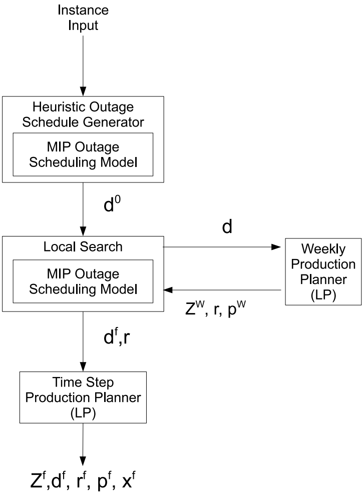
Conclusions



Approach:

- modular approach based on mathematical programming
- combinatorial part:
  - initial solution generated by constructive heuristic + MIP
  - local search + MIP for solution improvement
- continuous part:
  - quick weekly evaluator (LP)
  - time step final module:
    - one scenario at a time
    - only modulations
    - post-process solution to calculate refuellings, stocks (and the objective function. . .)

Part IV

Appendix

MIP Outage scheduling model

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Decision Variables

- $d_{i,k,h} \in \mathbb{B} = 1$ if plant i begins the k-th outage in week h. $\forall i, k, h : N^L_{ik} \leq h \leq N^U_{ik}$. It is the *decoupling date*
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- $x^A_{ik} \in \mathbb{R}_+$ is the initial stock of fuel in plant i at the beginning of cycle k, right *after* cycle $k-1$
- $r_{ik} \in \mathbb{R}_+$ is the *reload*

MIP Outage scheduling model: formulation (1/2)

(1) $R^{\min}_{ik} \cdot (1 - d_{ikH}) \leq r_{ik} \leq R^{\max}_{ik} \cdot (1 - d_{ikH}) \quad \forall i, k$

(2) $x^A_{i,-1} = X^0_i \quad \forall i$

(3) $x^B_{ik} \geq x^A_{i,k-1} - \sum_{h=0}^{H-1} d_{ikh} \cdot \mathrm{P}^{\max}_{ih} + \sum_{h=0}^{H-1-DAik} d_{i,k-1,h} \cdot \mathrm{P}^{\max}_{i,h+DA_{i,k-1}} - d_{ikH} \cdot M + d_{i,k-1,H} \cdot M \quad \forall i, k$

(4) $x^A_{ik} = R^r_{ik} \cdot (x^B_{ik} - BO_{i,k-1}) + r_{ik} + BO_{i,k-1} \quad \forall i, k$

(5) $x^B_{ik} \leq AMAX_{i,k} \quad \forall i, k$

(6) $x^A_{ik} \leq SMAX_{i,k} \quad \forall i, k$

(7) $\sum_{h=0}^{H} d_{ikh} = 1 \quad \forall i, k$

(8) $\sum_{k=0}^{K-1} d_{ikh} \leq 1 \quad \forall i, h$

(9) $d_{ikh} + \sum_{h'=h}^{\min(H-1, h+DA_{ik}+\mathrm{Se}_f)} d_{i'k'h'} \leq 1 \quad \forall f, i, i', k, h : f \in \mathrm{CT}_{14} \wedge i, i' \in A_f \wedge i \neq i'$

MIP Outage scheduling model: formulation (2/2)

$$(10)\quad d_{ikh} + \sum_{h'=h}^{\min(H-1,h+DA_{ik}+\mathrm{Se}_f-1,F_f)} \sum_{k'=0}^{K-1} d_{i'k'h'} \leq 1 \quad \begin{cases} \forall f,i,i',k,h : f \in \mathrm{CT}_{15} \wedge i,i' \in A_f \wedge i \neq i' \\ \wedge\, h = \max(0, I_f - DA_{ik}+1),\ldots,\min(F_f, H-1) \end{cases}$$

$$(11)\quad d_{ikh} + \sum_{h=h'}^{\min(H-1,h+\mathrm{Se}_f)} \sum_{k'=0}^{K-1} d_{i'k'h'} \leq 1 \quad \forall f,i,i',h,k : f \in \mathrm{CT}_{16} \wedge i,i' \in A_f \wedge i \neq i'$$

$$(12)\quad d_{ikh} + \sum_{k'=0}^{K-1} \sum_{h'=\max(h,h+DA_{ik}-\mathrm{Se}_f+1)}^{\min(H-1,h+DA_{ik}+\mathrm{Se}_f-1)} d_{i'k'h'} \leq 1 \quad \forall f,i,i',k,h : f \in \mathrm{CT}_{17} \wedge i,i' \in A_f \wedge i \neq i'$$

$$(13)\quad d_{ikh} + \sum_{k'=0}^{K-1} \sum_{h'=\max(h,h+DA_{ik}-\mathrm{Se}_f+1)}^{\min(H-1,h+DA_{ik}+\mathrm{Se}_f-1)} d_{i'k'h'} \leq 1 \quad \forall f,i,i',k,h : f \in \mathrm{CT}_{18} \wedge i,i' \in A_f \wedge i \neq i'$$

$$(14)\quad \sum_{i\in A_f} \sum_{k=0}^{K-1} \sum_{e=0}^{\min(h-\mathrm{ST}_{ikf},\mathrm{TU}_{ikf})} d_{ik,h-\mathrm{ST}_{ikf}-e} \leq Q_f \quad \forall f,h : f \in \mathrm{CT}_{19}$$

$$(15)\quad \sum_{i\in A_f(h)} \sum_{k=0}^{K-1} \sum_{h'=\max(0,h-DA_{ik}+1)}^{\min(h,H-1)} d_{ikh'} \leq E_{fh} \quad \forall f,h : f \in \mathrm{CT}_{20}$$

$$(16)\quad \sum_{i\in C_f} \sum_{k=0}^{K-1} \sum_{h'=\max(0,h-DA_{ik}+1)}^{h} d_{ikh'} \cdot P_{it}^{\max} \leq O_f^{\max} \quad \forall f,h,t : f \in \mathrm{CT}_{21} \wedge h \in \mathbb{W}_t \wedge t \in W_h$$

Weekly Production Planner (WPP)

Sets and Indices

- $j \in \mathbb{J} \triangleq \{0, \ldots, J-1\}$ is the index of type 1 power plants
- $s \in \mathbb{S} \triangleq \{0, \ldots, S-1\}$ is the index of scenarios
- $i \in \mathbb{I} \triangleq \{0, \ldots, I-1\}$ is the index of type 2 power plants
- $k \in \mathbb{K} \triangleq \{-1, 0, \ldots, K-1\}$ is the index of cycles
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- p^2_{ihs} is the power output of type 2 power plant i
- $x^B_{ik} \in \mathbb{R}_+$ is the remaining stock of fuel in plant i at the end of cycle $k-1$
- $x^A_{ik} \in \mathbb{R}_+$ is the initial stock of fuel in plant i at the beginning of cycle k
- $r_{ik} \in \mathbb{R}_+$ is the refuelling of plant i in cycle k
- x^f_{is} is the final fuel stock

a LP for the Weekly Production Planner (1/2)

(17) $$\min \quad \frac{1}{|S|} \sum_{s \in \mathbb{S}} \left[\sum_{h \in \mathbb{H}} \left(p^1_{hs} \cdot L^{\mathbb{H}}_h \cdot C^1_{hs} \right) - \sum_{i \in \mathbb{I}} C_{iH} \cdot x^f_{is} \right] + \sum_{i \in \mathbb{I}} \sum_{k \in \mathbb{K}} C_{ik} \cdot r_{ik}$$

subject to:

(18) $$p^1_{hs} + \sum_{i \in \mathbb{I}} p^2_{ihs} = DEM_{hs} \quad \forall h, s$$

(19) $$P1^{\min}_{hs} \leq p^1_{hs} \leq P1^{\max}_{hs} \quad \forall h, s$$

(20) $$p^2_{ihs} = 0 \quad \forall i, k, h : h \in \left[ha(i,k), ha(i,k) + DA_{ik} \right]$$

(21) $$p^2_{ihs} \leq P^{\max}_{ih} \quad \forall i, k, h : h \notin \left[ha(i,k), ha(i,k) + DA_{ik} \right]$$

(22) $$R^{min}_{ik} \leq r_{ik} \leq R^{\max}_{ik} \quad \forall i, k : ha(i,k) \neq -1$$

(23) $$x^A_{i,-1} = X^0_i \quad \forall i$$

(24) $$x^f_i = x^B_{i,k^*_i} \quad \forall i$$

(25)

a LP for the Weekly Production Planner (2/2)

$$x^B_{iks} = x^A_{i,k-1,s} - \sum_{h=ha(i,k-1)+DA_{i,k-1}}^{ha(i,k)} p^2_{ihs} \cdot L^{\mathbb{H}}_h \quad \forall i,\, k,\, s \tag{26}$$

$$x^A_{iks} = R^r_{ik} \cdot \left(x^B_{iks} - BO_{i,k-1}\right) + r_{ik} + BO_{ik} \quad \forall i,\, k,\, s \tag{27}$$

$$x^B_{iks} \leq AMAX_{ik} \quad \forall i,\, k,\, s \tag{28}$$

$$x^A_{iks} \leq SMAX_{ik} \quad \forall i,\, k,\, s \tag{29}$$

$$\sum_{h=ha(i,k)}^{N^{\min}_{ik}} \left[P^{\max}_{ih} - p^2_{ihs} \cdot L^{\mathbb{H}}_h\right] \leq MMAX_{ik} \quad \forall i,\, k,\, s \tag{30}$$

$$p^2_{ihs} = 0 \quad \forall i,\, k,\, h,\, s : N^{\max}_{ik} \leq h \leq ha(i,\, k) + DA_{ik} \tag{31}$$

Time Step Production Planner (WPP)

Sets and Indices

- $j \in \mathbb{J} \triangleq \{0, \ldots, J-1\}$ is the index of type 1 power plants
- $i \in \mathbb{I} \triangleq \{0, \ldots, I-1\}$ is the index of type 2 power plants
- $k \in \mathbb{K} \triangleq \{-1, 0, \ldots, K-1\}$ is the index of cycles
- $t \in \mathbb{T} \triangleq \{0, \ldots, T-1\}$ is the index of steps
- $h \in \mathbb{W} \triangleq \{0, \ldots, H\}$ is the index of weeks
- ($s \in \mathbb{S} \triangleq \{0, \ldots, S-1\}$ *is the index of scenarios*)

Note: one scenario at a time!

Decision Variables

- $p^{1s}_{jt} \in \mathbb{R}_+$, the production of type 1 power plant j in time step t in scenario s
- $p^{2s}_{it} \in \mathbb{R}_+$, the production of type 2 power plant i in time step t in scenario s
- $\Delta^+_{ik} \in \mathbb{R}_+$, the deviational variable for positive violation of the reference modulation for plant i in scenario k
- $\Delta^-_{ik} \in \mathbb{R}_+$, the deviational variable for negative violation of the reference modulation for plant i in scenario k

a LP for the Time Step Production Planner

(32) $$\min \frac{1}{S} \sum_{j} \sum_{t} \left[C_{jts} \cdot L_t^{\mathbb{T}} \cdot p_{jt}^{1s} \right] + M \cdot \sum_{i} \sum_{k} \left[\Delta_{ik}^{+} + \Delta_{ik}^{-} \right]$$

subject to:

(33) $$\sum_{j \in \mathbb{J}} p_{jt}^{1s} + \sum_{i \in \mathbb{I}} p_{it}^{2s} = DEM^{ts} \quad \forall t$$

(34) $$P_{jts}^{\min} \leq p_{jt}^{1s} \leq P_{jts}^{\max} \quad \forall t, j$$

(35) $$p_{it}^{2s} = 0 \quad \forall i, k, t : t \in \mathbb{T}_{ik}^{O}$$

(36) $$p_{it}^{2s} \leq P_{it}^{\max} \quad \forall t, i, k : t \in \mathbb{T}_{ik}^{P}$$

(37) $$\sum_{t \in \mathbb{T}_{ik}^{P}} p_{it}^{2s} \cdot L_t^{\mathbb{T}} + \Delta_{ik}^{+} - \Delta_{ik}^{-} = \sum_{t \in \mathbb{T}_{ik}^{P}} P_{ik}^{\max} \cdot L_t^{\mathbb{T}} - \mathrm{mod}_{ik}^{\max} \quad \forall i, k$$

(38) $$\Delta^{+} \leq \max \left(MMAX_{ik} - \mathrm{mod}_{ik}^{\max}, 0 \right) \quad \forall i, k$$

(39) $$\Delta^{-} \leq MMAX_{ik} \quad \forall i, k$$

(40) $$p_{it}^{2s} = \mathrm{prof}\left(x_{its} \right) \cdot P_{ik}^{\max} \quad \forall i, k, t : t \in \mathbb{T}_{ik}^{P} \wedge x_{it} < BO_{ik}$$