FAPP – Frequency Assignment Problem with Polarization.

ROADEF CHALLENGE 2001[1]

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Summary :

The ever-increasing demand for communication, coupled with the limit of spectra available, have made frequency assignment more and more difficult to accomplish effectively. Optimization of this process has therefore become a preoccupation for both civil and military network administration and deployment.

One role of the CELAR, a technical center of Délégation Générale pour l'Armement (DGA), is to study new methods to optimize the use of the available spectra for the French army. The subject proposed here concerns, in particular, frequency allocation in Hertzian telecommunication networks. This problem, studied within the project CALMA (Combinatorial ALgorithms for Military Applications) from 1993-95 was further enriched to take into account polarization, and the relaxation of certain electromagnetic compatibility constraints.

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[1] principal URL (ROADEF) : http://www.prism.uvsq.fr/~vdc/ROADEF/CALLENGES/2001/

secondary URL (ENSTA) : http://www.ensta.fr/~diam/roadef2001_fapp

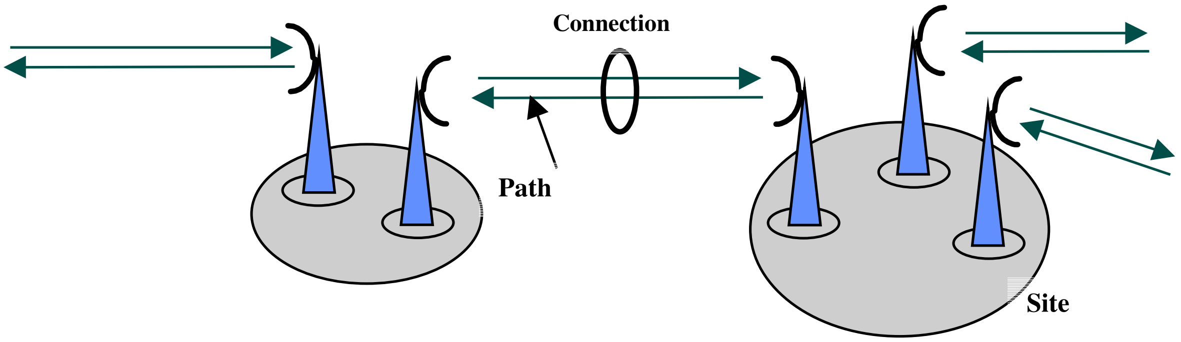
[2] CELAR/TCOM – BP 7419 – 35174 BRUZ Cedex

1 The probleme

1.1 'Physical' description

The problem presented here is that of the optimal frequency allocation in hertzian telecommunication networks. The network is therefore composed of a set of **sites** in which the transmission supports (antennas connected to emitters or receptors) are located. A hertzian **connection** joins two geographic sites; it may be composed of one or more **paths**.

A **path** is a uni-directional radio-electric bond, established between antennas at distinct sites, and having a given frequency and polarization. We will refer to the set of paths for a problem as T.



We define a **frequency resource** as a pair (**frequency**, **polarization**) in which the components are associated with the carrying frequency of the transmitted signal, and to the wave polarization, respectively. To simplify the model, we will consider the latter to be a binary variable (e.g., only vertical or horizontal).

For each path $i \in T$, we define a set of resources available, called the **domain** of the path. It is composed on the one hand of the domain frequency, referred to as $F_{\varphi(i)}$, and on the other hand of the polarization information, $P_{\pi(i)}$ which may include a required polarization or not. Two paths may have identical domains.

A frequency allocation consists of obtaining a pair (frequency, polarization) for each path and satisfying certain radio-electric compatibility constraints.

In addition to the abovementioned constraints, several coupling constraints must be taken into account. They are:

1. Equality or not of frequencies across paths: $f_i = f_j$ or $f_i \neq f_j$,
2. Distance between frequencies: $\left| f_i - f_j \right| = \varepsilon_{ij}$ or $\left| f_i - f_j \right| \neq \varepsilon_{ij}$,
3. Equality or not of polarities across paths: $p_i = p_j$ or $p_i \neq p_j$,
4. Minimal distance between frequencies: $\left| f_i - f_j \right| \geq \begin{cases} \gamma_{ij} & \text{si } p_i = p_j \\ \delta_{ij} & \text{si } p_i \neq p_j \end{cases}$.

Constraint 4 above requires that the required distance between frequencies depend on their polarizations. In general, the minimal distance is smaller if the polarizations are not identical.

We define a **feasible solution** to be an allocation for each path that satisfies the full set of constraints.

Unfortunately, most problems do not admit any such feasible solution. The domains are too restrictive or the requirements to numerous. The operator must therefore look for "partial" solution. For this purpose, we introduce two classes of constraints:

- Class 1: Strong or necessary constraints (1-3 above, and domain constraints)
- Class 2: CEM constraints (type 4) for which we will authorize a progressive relaxation. For this purpose, we introduce the relaxation level: level 0 corresponds to no relaxation. Going from level k to level k+1 involves a relaxation of some or all of the frequency distances defining each constraint, the maximum level being 10:

$$\left|f_i - f_j\right| \geq \begin{cases} \gamma_{ij}^{(0)} \geq \gamma_{ij}^{(1)} \geq \ldots \geq \gamma_{ij}^{(k)} \geq \ldots \geq \gamma_{ij}^{(10)} & \text{si } p_i = p_j \\ \delta_{ij}^{(0)} \geq \delta_{ij}^{(1)} \geq \ldots \geq \delta_{ij}^{(k)} \geq \ldots \geq \delta_{ij}^{(10)} & \text{si } p_i \neq p_j \end{cases}$$

In this context, we will define a **feasible solution of level k** to be an allocation for each path that satisfies all the strong and necessary constraints and all constraints at level k. We call such a problem ***k*-feasible.**

In case no feasible solution exists, the new objective function becomes, in order of priority :

1. search for k^* the smallest relaxation level for which a k^*-feasible solution exists;
2. minimize the number of CEM constraints not satisfied at level $k^* - 1$;
3. minimize the number of CEM constraints not satisfied at levels less than $k^* - 1$.

1.2 Mathematical formulation

Let T be the set of paths, $F_0, F_1, \ldots, F_N : N$ the frequency domains (in general, N << |T|).

Let $P_{-1}=\{-1\}$, $P_1=\{1\}$, $P_0=\{-1,1\}$ be the three possible polarization domains.

For each $i \in T$, we associate : $\begin{cases} f_i \in F_{\varphi(i)} \\ p_i \in P_{\pi(i)} \end{cases}$

where φ(i) and π(i) indicate respectively the frequency and polarization domain numbers of *i*.

In matrix form, the constraints are :

$$
\begin{cases}
|f_i - f_j| \geq \frac{|p_i + p_j|}{2}\gamma_{ij}^{(k)} + \frac{|p_i - p_j|}{2}\delta_{ij}^{(k)}, k = 0,1,\ldots,10 \\
|f_i - f_j| = \varepsilon_{ij} \\
|f_i - f_j| \neq \varepsilon_{ij} \\
f_i = f_j \\
f_i \neq f_j \\
p_i = p_j \\
p_i \neq p_j
\end{cases}
$$

Let $\Theta = \left|\left\{(i,j) : \gamma_{ij}^{(0)} + \delta_{ij}^{(0)} > 0\right\}\right|$, the number of CEM (class 2) constraints

Let $V^{(k)} = \left\{(i,j) : |f_i - f_j| < \frac{|p_i + p_j|}{2}\gamma_{ij}^{(k)} + \frac{|p_i - p_j|}{2}\delta_{ij}^{(k)}\right\}$, the number of violated constraints at level k.

We set $V^{(-1)} = \varnothing$ et every problem is 11-feasible.

We would like to minimize the following function , **c(T,F,P,M)**:

$\boxed{c(T,F,P,M) = \left(10\Theta^2 k + 10\Theta\left|V^{(k-1)}\right| + \left|\bigcup_{i<k-1} V^{(i)}\right|\right)}$ where k is such that the problem is k-feasible.

<u>Remark :</u> If a constraint is violated at level m, it is violated at j<m, and therefore counted m+1 times in the cost function.

2 File formats

The input files (name.in) and the output files (name.out) are ASCII files. Each input file contains data for the allocation model, and each output file contains solution evaluation data. All data are formatted according to the rules below.

- Numerical fields are aligned to the right
- Alphanumerical fields are aligned to the left
- A field of the type ‘CEM with equal polarizations’ must always be followed by a field of the type ‘CEM with different polarizations’ for a single path pair.

The different fields are:

- Input file
    - Domain,
    - Path,
    - Class 1 constraints
    - Class 2 constraints with equal and cross polarisations
- Output file
    - Results,
    - Assignment

2.1 Input file

2.1.1 General

In the input file, the field order is Domain, Path, Constraints. Integer fields contain 5or 9 characters aligned to the right, so that 33 has 3 spaces before it (illustrated with a period): `...33`.

2.1.2 ‘Domain’ field:

TypRec = DM

TypRec			Domain number (integer)						Frequency (integer)				
D	M		#	#	#	#	#		#	#	#	#	#

In the C-language, one writes : `"DM.%5d.%5d\n"`

Example :

For the domain : F_0={10,11,12,13, 14,15}, we have :

```
DM.....0....10
DM.....0....11
DM.....0....12
DM.....0....13
DM.....0....14
DM.....0....15
```

2.1.3 Path field

TypRec = TR

TypRec			Path Number (Integer)						Frequency domain Number (Integer)						Polarization domain Number		
T	R		#	#	#	#	#		#	#	#	#	#		#	#	

In C : `"TR.%5d.%5d.%2d\n"`

<u>Reminder:</u> The three polarisations possible are : $P_{-1}=\{-1\}$, $P_1=\{1\}$, $P_0=\{-1,1\}$.

<u>**Example :**</u>

Problem with four paths:

Path	φ(i)	π(i)
1	0	-1
2	0	0
3	14	0
4	5	1

The input file includes :

```
TR.....1.....0.-1
TR.....2.....0..0
TR.....3....14..0
TR.....4.....5..1
```

2.1.4 'Class 1 constraint'

TypRec = CI

TypRec			Number of path 1 (Integer)						Number of path 2 (Integer)						C1		C2		Distance valuet (Integer)				
C	I		#	#	#	#	#		#	#	#	#	#		#		#		#	#	#	#	#

In C: `"CI.%5d.%5d.%c.%c.%5d\n"`

Field C1 :

C1 = F if the constraint concerns frequencies
C1 = P if the constraint concerns polarisations

Field C2 :

C2 = E for an equality constraint
C2 = I for an inequality constraint

Examples :

$f_{12} = f_{19}$	`CI....12....19.F.E.....0`
$f_{45} \neq f_{184}$,	`CI....45...184.F.I.....0`
$\lvert f_5 - f_{1498} \rvert = 40$	`CI.....5..1498.F.E....40`
$\lvert f_9 - f_{134} \rvert \neq 10$	`CI.....9...134.F.I....10`
$p_6 = p_9$	`CI.....6.....9.P.E.....0`
$p_{892} \neq p_{52}$	`CI...892....52.P.I.....0`

2.1.5 'Equal polarisation' class 2 constraint field

TypRec = CE

TypRec			Numberof path 1 (Integer)						Number of path 2 (Integer)						$\gamma_{ij}^{(0)}$ (Integer)						…		$\gamma_{ij}^{(10)}$ (Integer)				
C	E		#	#	#	#	#		#	#	#	#	#		#	#	#	#	#		…		#	#	#	#	#

In C:

```
"CE.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d\n".
```

Examples :

Let $p_{34}=p_{16}$ the constraint is:

$$\left|f_{34}-f_{16}\right| \geq 45 \geq 45 \geq 44 \geq 43 \geq 40 \geq 40 \geq 40 \geq 40 \geq 40 \geq 35 \geq 30$$

expressed as (on one line):

```
CE....34....16....45....45....44....43....40....40....40....40....40....35....30
```

2.1.6 'Unequal polarisation' class 2 constraint field

TypRec = CD

TypRec			Number of path 1 (Integer)						Number of path 2 (Integer)						$\delta_{ij}^{(0)}$ (Integer)						…		$\delta_{ij}^{(10)}$ (Integer)				
C	D		#	#	#	#	#		#	#	#	#	#		#	#	#	#	#		…		#	#	#	#	#

In C :

```
"CD.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d.%5d\n".
```

Examples :

For the constraint $p_{145} \neq p_{84}$:

$$\left|f_{145}-f_{84}\right| \geq 203 \geq 200 \geq 200 \geq 150 \geq 150 \geq 125 \geq 125 \geq 100 \geq 99 \geq 95 \geq 90$$

we write:

```
CD...145....84...203...200...200...150...150...125...125...100....99....95....90
```

2.2 Output file

2.2.1 Reponse field

TypRec = RP

TyrRec		k			C1		Time to obtain the best value for k in seconds						Time to obtain the optimality guarantee for k in seconds				
R			#		#		#	#	#	#	#		#	#	#	#	#

• • •

$\left\|V^{(k-1)}\right\|$										C2		Time to obtain the best value for $\left\|V^{(k-1)}\right\|$ in seconds						Time to obtain the optimality guarantee for $\left\|V^{(k-1)}\right\|$ in seconds				
	#	#	#	#	#	#	#	#		#		#	#	#	#	#		#	#	#	#	#

• • •

$\left\|\bigcup_{i<k-1} V^{(i)}\right\|$										C3		Time to obtain the best value for $\left\|\bigcup_{i<k-1} V^{(i)}\right\|$ in seconds						Time to obtain the optimality guarantee for $\left\|\bigcup_{i<k-1} V^{(i)}\right\|$ in seconds						Total time Ttot				
		#	#	#	#	#	#	#		#		#	#	#	#	#		#	#	#	#	#		#	#	#	#	#

In C :

```
"RP.%2d.%c.%5d.%5d.%9d.%c.%5d.%5d.%9d.%c.%5d.%5d %5d\n".
```

Field C1 :
- C1 = 1 if optimality is obtained at k
- C1 = 0 otherwise

Field C2 :
- C2 = 1 if the value $\left|V^{(k-1)}\right|$ is obtained at optimality
- C2 = 0 otherwise

Field C3 :
- C3 = 1 if the value $\left|\bigcup_{i<k-1} V^{(i)}\right|$ is obtained at optimality
- C3 = 0 otherwise

Remarks:

The times T1, T2, and T3 gives the time taken to obtain the best value of the associated value.

Ttot indicates the total processor time (Ttot=3600 sec if the program is stopped after one hour, or Ttot<3600 if the optimal solution is obtained earlier).

If the value is not known, then Ttot=99999. (See the examples).

2.2.2 Assignment field

TypRec = AL

TypRec			Number of path (Integer)						Frequence allouee (Integer)						Polar. (Integer)	
A	L		#	#	#	#	#		#	#	#	#	#		#	#

In C : `"AL.%5d.%5d.%2d\n"`

Given a problem with four paths :

Path	f_i	p_i
1	10	-1
2	2	1
3	18	1
4	40	1

The file name.out contains :

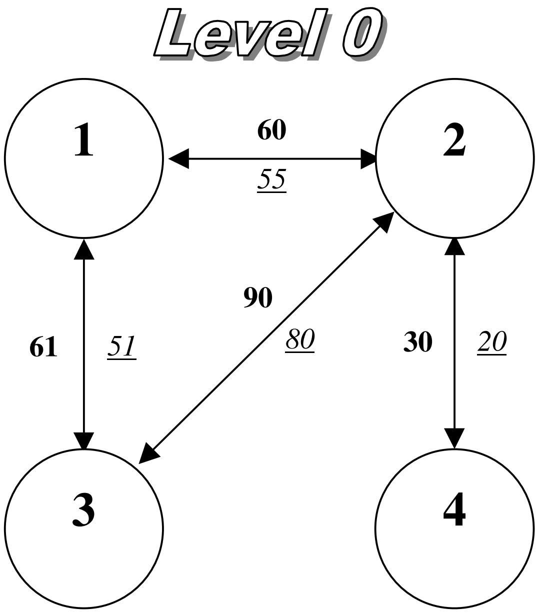
```
AL.....1....10.-1
AL.....2.....2..1
AL.....3....18..1
AL.....4....40..1
```

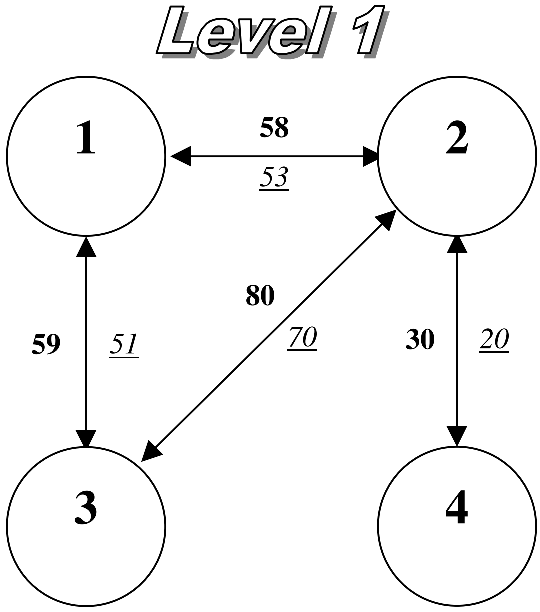
3 Examples

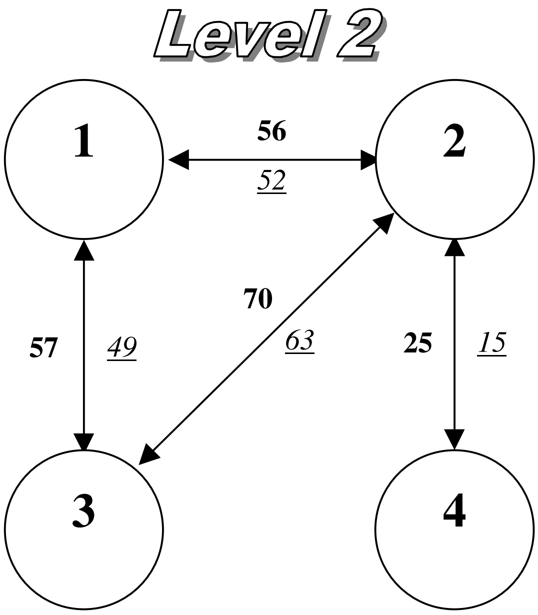
3.1 Example 1 – unique domain frequency without class 1 constraints

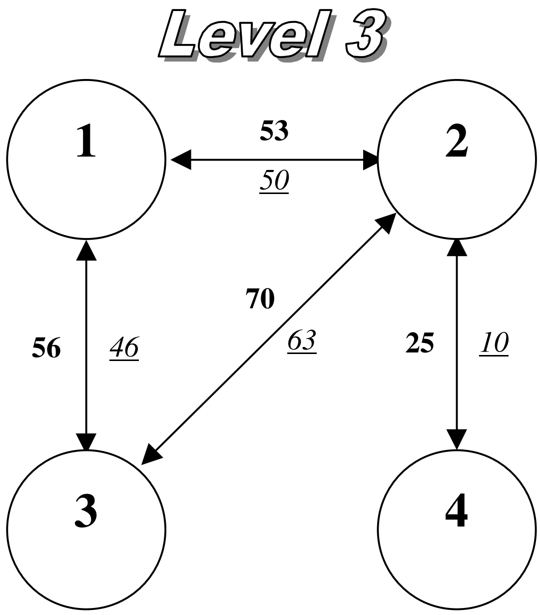
The four paths, numbered 1-4, share the same domain frequency and polarization: for all i=1,2,3,4, $\varphi(i)=\pi(i)=0$ with $F_0=\{1,2,\ldots,100\}$.

The 11 figures below illustrate the constraints at each level. The CEM identical polarization constraint is indicated in **bold** while the cross polarization CEM constraint is illustrated in <u>*underlined italics*</u>. In this example, only CEM constraints are imposed; there are no class 1 constraints.

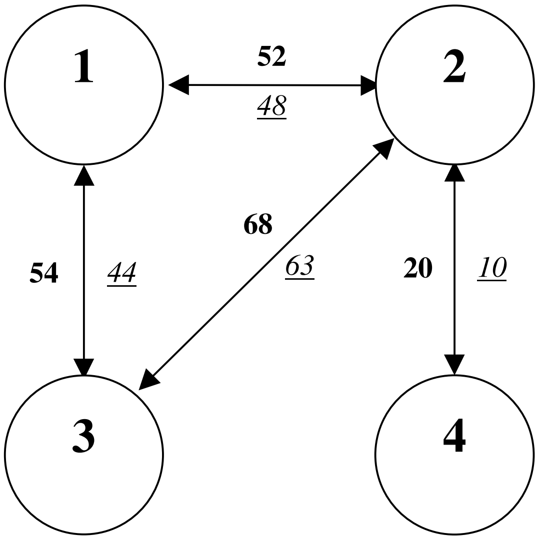




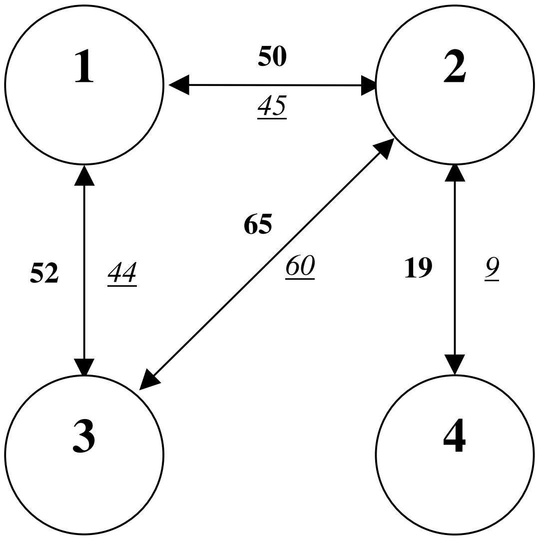




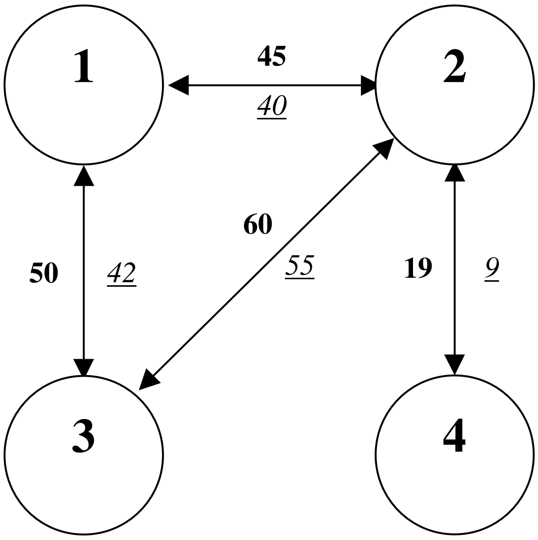
Level 4



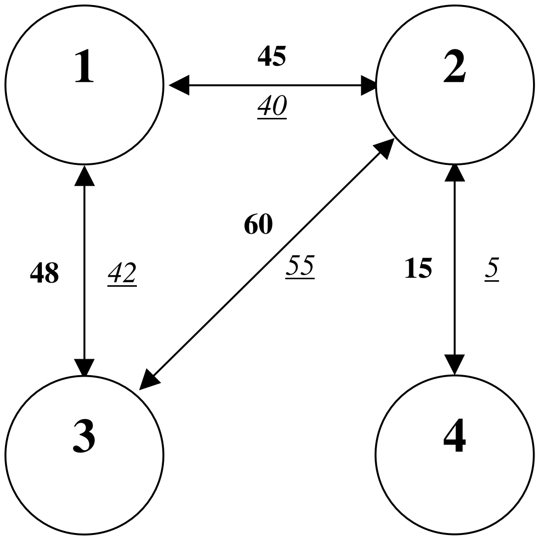
Levels 5 and 6



Levels 7 to 9



Level 10



The file Example1.in looks like :

```
DM.....0.....1
DM.....0.....2
DM.....0.....3
```

There are as many DM lines as values, 100 in all.

```
DM.....0...100
TR.....1.....0..0
TR.....2.....0..0
TR.....3.....0..0
TR.....4.....0..0
CE.....1.....2....60....58....56....53....52....50....50....45....45....45....45
CD.....1.....2....55....53....52....50....48....45....45....40....40....40....40
CE.....1.....3....61....59....57....56....54....52....52....50....50....50....48
CD.....1.....3....51....51....49....46....44....44....44....42....42....42....42
CE.....2.....3....90....80....70....70....68....65....65....60....60....60....60
CD.....2.....3....80....70....63....63....64....60....60....55....55....55....55
CE.....2.....4....30....30....25....25....20....19....19....19....19....19....15
CD.....2.....4....20....20....15....10....10.....9.....9.....9.....9.....9.....5
```

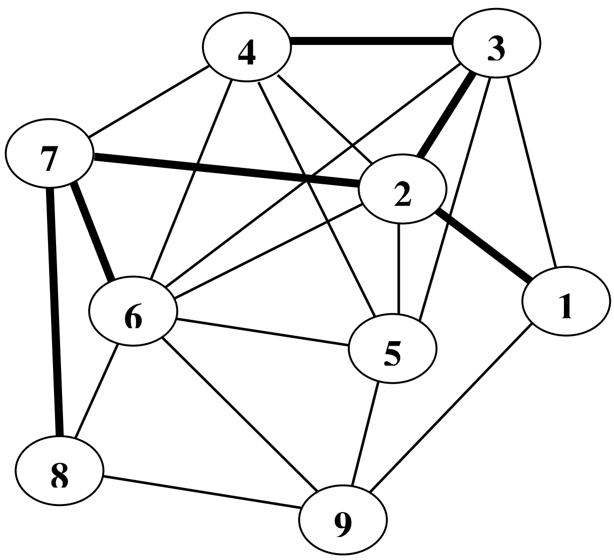
The file Example1.out may look like :

```
RP..3.1...500...750.........1.1..1800..1800.........4.0..3590.99999..3600
AL.....1....51..1
AL.....2.....1.-1
AL.....3...100.-1
AL.....4....40.-1
```

We have obtained the first 3-feasible (k=3) solution in 500 sec, and the optimality proof for the value of k was obtained at 750 sec. Then, we found a 3-feasible solution with one non-satisfied constraint at level 3-1=2 in 1800 sec and the optimality proof of that number of violated constraints at 3000 sec. Finally, a 3-feasible solution having 4 constraints unsatisfied for levels 0-1 was found in 3590 sec, but the optimality proof was not found, giving a total time of 3600.

3.2 Example 2 - complete

We consider a problem with 9 paths. The following figure illustrates the constraints imposed across paths. A thick line indicates a class 1 constraint while a thin line indicates a CEM (class 2) constraint.



We have three domain frequencies :

$F_0=\{0,1,2,3,\ldots,99,100\}$

$F_1=\{25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40\}$

$F_2=\{55,56,57,63,64,65\}$

The paths, numbered 1-9, have the following resources :

Path	φ(i)	π(i)
1	1	-1
2	1	0
3	0	0
4	2	1
5	0	0
6	0	0
7	2	0
8	0	0
9	0	0

The constraints are :

1. Class 1 constraints
   - $|f_2 - f_3| = 36$
   - $f_1 = f_2$
   - $f_7 \neq f_8$
   - $p_3 = p_4$
   - $p_2 = p_7$
   - $p_6 \neq p_7$
2. CEM constraints
   - $\gamma_{13} = (46,44,42,42,40,40,35,35,35,30,20)$ $\delta_{13} = (39,37,35,35,33,33,28,28,28,23,13)$
   - $\gamma_{19} = (30,29,28,27,26,25,24,23,22,21,20)$ $\delta_{19} = (29,28,27,26,25,24,23,22,21,20,19)$
   - $\gamma_{24} = (33,32,30,30,29,28,27,22,17,12,7)$ $\delta_{24} = (29,28,27,27,27,26,25,20,15,10,5)$
   - $\gamma_{25} = (80,78,76,74,74,72,70,45,40,35,30)$ $\delta_{25} = (75,73,71,69,69,67,65,40,35,30,25)$
   - $\gamma_{26} = (55,53,50,45,40,40,40,35,30,25,20)$ $\delta_{26} = (50,48,45,40,36,36,36,30,25,20,15)$
   - $\gamma_{35} = (20,20,20,20,20,20,20,20,20,20,20)$ $\delta_{35} = (19,19,19,19,19,19,19,19,19,19,19)$

- $\gamma_{36} = (50,48,46,44,42,40,38,36,34,32,30)$
  $\delta_{36} = (50,48,46,44,42,40,38,36,34,32,30)$
- $\gamma_{45} = (30,27,26,25,22,21,19,16,15,13,10)$
  $\delta_{45} = (29,25,25,23,21,19,18,14,14,11,9)$
- $\gamma_{47} = (15,14,12,9,5,0,0,0,0,0,0)$
  $\delta_{47} = (10,9,7,4,0,0,0,0,0,0,0)$
- $\gamma_{56} = (55,48,45,43,43,41,41,41,40,40,40)$
  $\delta_{56} = (50,47,43,42,42,41,40,39,39,39,35)$
- $\gamma_{59} = (30,30,30,27,27,27,23,23,23,20,20)$
  $\delta_{59} = (25,25,25,22,22,22,20,20,20,15,15)$
- $\gamma_{68} = (50,48,44,42,38,36,32,30,26,24,20)$
  $\delta_{68} = (47,45,41,39,35,33,29,27,23,21,17)$
- $\gamma_{69} = (13,13,13,13,13,13,13,5,5,5,0)$
  $\delta_{69} = (10,10,10,10,10,10,10,1,1,1,0)$
- $\gamma_{89} = (20,19,17,15,14,12,11,9,8,6,5)$
  $\delta_{89} = (15,14,12,11,9,8,6,5,3,2,0)$

The file Example2.in is :

```
DM.....0.....0
DM.....0.....1
DM.....0.....2
```

101 lines DM, as many as values

```
DM.....0...100
DM.....1....25
DM.....1....26
DM.....1....27
DM.....1....28
DM.....1....29
DM.....1....30
DM.....1....31
DM.....1....32
DM.....1....33
DM.....1....34
DM.....1....35
DM.....1....36
DM.....1....37
DM.....1....38
DM.....1....39
DM.....1....40
DM.....2....55
DM.....2....56
DM.....2....57
DM.....2....63
DM.....2....64
DM.....2....65
TR.....1.....1.-1
TR.....2.....1..0
```

```
TR.....3.....0..0
TR.....4.....2..1
TR.....5.....0..0
TR.....6.....0..0
TR.....7.....2..0
TR.....8.....0..0
TR.....9.....0..0
CI.....2.....3.F.E....36
CI.....1.....2.F.E.....0
CI.....7.....8.F.I.....0
CI.....3.....4.P.E.....0
CI.....2.....7.P.E.....0
CI.....6.....7.P.I.....0
CE.....1.....3....46....44....42....42....40....40....35....35....35....30....20
CD.....1.....3....39....37....35....35....33....33....28....28....28....23....13
CE.....1.....9....30....29....28....27....26....25....24....23....22....21....20
CD.....1.....9....29....28....27....26....25....24....23....22....21....20....19
CE.....2.....4....33....32....30....30....29....28....27....22....17....12.....7
CD.....2.....4....29....28....27....27....27....26....25....20....15....10.....5
CE.....2.....5....80....78....76....74....74....72....70....45....40....35....30
CD.....2.....5....75....73....71....69....69....67....65....40....35....30....25
CE.....2.....6....55....53....50....45....40....40....40....35....30....25....20
CD.....2.....6....50....48....45....40....36....36....36....30....25....20....15
CE.....3.....5....20....20....20....20....20....20....20....20....20....20....20
CD.....3.....5....19....19....19....19....19....19....19....19....19....19....19
CE.....3.....6....50....48....46....44....42....40....38....36....34....32....30
CD.....3.....6....50....48....46....44....42....40....38....36....34....32....30
CE.....4.....5....30....27....26....25....22....21....19....16....15....13....10
CD.....4.....5....29....25....25....23....21....19....18....14....14....11.....9
CE.....4.....7....15....14....12.....9.....5.....0.....0.....0.....0.....0.....0
CD.....4.....7....10.....9.....7.....4.....0.....0.....0.....0.....0.....0.....0
CE.....5.....6....55....48....45....43....43....41....41....41....40....40....40
CD.....5.....6....50....47....43....42....42....41....40....39....39....39....35
CE.....5.....9....30....30....30....27....27....27....23....23....23....20....20
CD.....5.....9....25....25....25....22....22....22....20....20....20....15....15
CE.....6.....8....50....48....44....42....38....36....32....30....26....24....20
CD.....6.....8....47....45....41....39....35....33....29....27....23....21....17
CE.....6.....9....13....13....13....13....13....13....13.....5.....5.....5.....0
CD.....6.....9....10....10....10....10....10....10....10.....1.....1.....1.....0
CE.....8.....9....20....19....17....15....14....12....11.....9.....8.....6.....5
CD.....8.....9....15....14....12....11.....9.....8.....6.....5.....3.....2.....0
```

The file Example2.out may look like :

```
RP..7.1..1200..1559.........1.1..2693..3562........14.0..3598.99999..3600
AL.....1....37.-1
AL.....2....37.-1
AL.....3....73..1
AL.....4....65..1
AL.....5....98..1
AL.....6.....1..1
AL.....7....56.-1
AL.....8....55..1
AL.....9....70.-1
```

The problem is 7-feasible. With the given assignment, we have one unsatisfied constraint at level 6. In addition, there are 14 unsatisfied constraints for levels 0-5. This solution is optimal for k and the number of violated constraints at lower levels, but not necessisarily for the last criterion.