Carbon-Constrained Lot-sizing

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Joint work with
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Outline

Introduction

- Context
  - Legislations are evolving in order to enforce control on carbon emissions.
  - Companies will face new constraints that will force them to reduce carbon emissions while still minimizing production and transportation costs.
  - Considering green logistics objectives and constraints lead to new lot-sizing problems

Literature review

- Few papers addressing production planning and transportation problems that take into account environmental constraints.
- Environmental constraints are integrated as cost components in the objective function (multi-criteria approaches) (see Handfield et al. (2002), Aissaoui et al. (2007), van den Heuvel et al. (2012)).
- The limit of these models is that classical cost components have the same behavior than environmental cost components (e.g. reducing the number of vehicles).
- Benjaafar et al. (T-ASE 2013) addresses the integration of carbon emission constraints in lot-sizing problems. Capacity constraint that links and limits all carbon emissions over the planning horizon related to production and storage.
- Helmrich et al. (2012) consider the previous global emission constraint, shows that the problem is NP-hard and propose a Lagrangian heuristic, a pseudo-polynomial algorithm and a FPTAS.

Contributions

- Different carbon emission constraints in lot-sizing models, complexity analysis (Absi et al. (EJOR 2013)).
- Analysis of periodic carbon emission constraints with fixed carbon emissions.
- Complexity analysis.

Lot-sizing problems

Absi et al. (EJOR 2013)

- Multi-sourcing Uncapacitated single-item Lot-Sizing (ULS) problems with carbon emission constraints
  - Planning horizon of T periods.
  - M supplying modes, combination of a production location and a transportation mode.

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  - Global: extends the cumulative constraint on the whole horizon.

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- Four types of carbon emission constraints
  - Periodic: the carbon emission limit is fixed for each period.
  - Cumulative: the amount of unused carbon emission of a given period can be reused.
  - Global: extends the cumulative constraint on the whole horizon.
  - Rolling: carbon emissions compensation between periods on a rolling horizon.
    The rolling carbon emission constraint includes the Periodic and the Global carbon emission constraints as special cases.

Lot-sizing problems

Variable Carbon Emission Parameter

Contributions

Mathematical programming models

Parameters

- d_t: Demand at period t.
- $h_t(s)$: Cost of holding s units at period t.
- p_t^m: Unitary supplying cost of mode m at period t.
- f_t^m: Supplying setup cost of mode m at period t.

Mathematical programming models

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- p_t^m: Unitary supplying cost of mode m at period t.
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- e_t^m: Environmental impact (carbon emission) related to supplying one unit using mode m at period t.
- $E_t^{\max}$: Maximum unitary environmental impact allowed at period t.

A mode m is called ecological in period t if $e_t^m \leq E_t^{\max}$.

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Variables

- x_t^m: Quantity supplied at period t using mode m.
- y_t^m: Binary variable equal to 1 if mode m is used at period t, and 0 otherwise.
- s_t: Inventory carried from period t to period $t+1$.

Mathematical model

Multi-sourcing uncapacitated single-item lot-sizing problems without carbon emission constraints

$$
\begin{aligned}
\min \quad & \sum_{m=1}^{M} \sum_{t=1}^{T} (p_t^m x_t^m + f_t^m y_t^m) + \sum_{t=1}^{T} h_t(s_t) \\
s.t. \quad & \sum_{m=1}^{M} x_t^m - s_t + s_{t-1} = d_t, \quad t = 1, \ldots, T \\
& x_t^m \le \left(\sum_{t'=t}^{T} d_{t'} \right) y_t^m, \quad t = 1, \ldots, T, m = 1, \ldots, M \\
& x_t^m \in \mathbb{R}^+, y_t^m \in \{0, 1\}, \quad t = 1, \ldots, T, m = 1, \ldots, M \\
& s_t \in \mathbb{R}^+, \quad t = 1, \ldots, T
\end{aligned}
$$

Carbon emission constraints

- Periodic

$$\frac{\sum_{m=1}^{M} e_t^m x_t^m}{\sum_{m=1}^{M} x_t^m} \leq E_t^{\max}, \; t = 1, \ldots, T.$$

$$\sum_{m=1}^{M} (e_t^m - E_t^{\max}) x_t^m \leq 0, \; t = 1, \ldots, T.$$

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- Cumulative

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- Cumulative

$$\sum_{t'=1}^{t} \sum_{m=1}^{M} (e_t^m - E_{t'}^{\max}) x_{t'}^m \leq 0, \;\; t = 1, \ldots, T.$$

- Global

$$\sum_{t=1}^{T} \sum_{m=1}^{M} (e_t^m - E^{\max}) x_t^m \leq 0.$$

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- Global

$$\sum_{t=1}^{T} \sum_{m=1}^{M} (e_t^m - E^{\max}) x_t^m \leq 0.$$

- Rolling

$$\sum_{t'=t-R+1}^{t} \sum_{m=1}^{M} (e_t^m - E_{t'}^{\max}) x_{t'}^m \leq 0, \; t = R, \ldots, T.$$

The single-item ULS problem with Periodic Carbon emission constraint ULS-PC

$$\sum_{m=1}^{M}(e^m - E_t^{\max})X_t^m \leq 0, \; t = 1, \ldots, T.$$

- Preliminary properties:
  - If $p_t^{m_1} \leq p_t^{m_2}$ and $e_t^{m_1} \leq e_t^{m_2}$, then mode m_1 dominates mode m_2.
  - Any solution of the ULS-PC problem uses at least one ecological mode in each period with an order.

Structural properties of the optimal solution for the ULS-PC (1)

Theorem 1

There exists an optimal solution for the ULS-PC problem that uses at most two modes in each period: One ecological mode and possibly one non-ecological mode.

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Sketch of the Proof

- Decomposition (Bender's approach) into a master problem (MP) and T independent $IP_t(X_t)$ where $X_t = \sum_m x_t^m$.
- F_t denotes the set of the ecological modes at period t.

$$
(MP)\left\{\begin{array}{lll}
\min & \sum_{t=1}^{T} z_t^*(X_t) + \sum_{t=1}^{T} h_t(s_t) & \\
s.t. & X_t - s_t + s_{t-1} = d_t & t = 1,\ldots,T \\
 & X_t = 0 & t = 1,\ldots,T \text{ such that } F_t = \emptyset \\
 & X_t \in \mathbb{R}^+ & t = 1,\ldots,T
\end{array}\right.
$$

where $z_t^*(X_t)$ is the optimal value of problem $IP_t(X_t)$ given by the following formulation:

$$
(IP_t(X_t))\left\{\begin{array}{lll}
\min & \sum_{m=1}^{M} (p_t^m x_t^m + f_t^m y_t^m) & \\
s.t. & \sum_{m=1}^{M} x_t^m = X_t & \\
 & \sum_{m=1}^{M} (e_t^m - E_t^{\max}) x_t^m \leq 0 & \\
 & x_t^m \leq X_t y_t^m & m = 1,\ldots,M \\
 & x_t^m \in \mathbb{Z}^+,\ y_t^m \in \{0,1\} & m = 1,\ldots,M
\end{array}\right.
$$

- Problem $IP_t(X_t)$ consists in supplying X_t in period t at minimum cost while satisfying a carbon emission constraint.
- $IP_t(X_t)$ problem is feasible if and only if at least an ecological mode is available (Constraints ($F_t = \emptyset \Rightarrow X_t = 0$) in (MP)).

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- 2 cases:
  - Relaxation in $IP_t(X_t)$ of constraint $x_t^m \leq X_t y_t^m$ (no setup cost): The problem reduces to a LP on variables x^m with only two constraints.

    From elementary LP theory, at most 2 variables are not null, one corresponding to an ecological mode and the other to a non-ecological mode.

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  - Relaxation in $IP_t(X_t)$ of constraint $x_t^m \leq X_t y_t^m$ (no setup cost): The problem reduces to a LP on variables x^m with only two constraints.
    From elementary LP theory, at most 2 variables are not null, one corresponding to an ecological mode and the other to a non-ecological mode.
  - Setup costs: Let $\hat{\pi}$ be a feasible policy and $\hat{\mathcal{M}}_t$ the subset of modes used in period t.
    We can easily transform $\hat{\pi}$ into a feasible policy of lower cost using at most 2 modes in each period by solving problem $IP_t(\hat{X}_t)$ where y_t are fixed to $\hat{y}_t$ (case 1 with a restricted subset of modes $\hat{\mathcal{M}}_t$).

Structural properties of the optimal solution for the ULS-PC (2)

Theorem 2

The ULS-PC problem can be reformulated as an uncapacitated multi-sourcing lot-sizing problem with M^2 modes using $O(M^2T)$ operations.

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Theorem 2

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Sketch of the Proof

- An optimal policy m_1, m_2 at period t, with m_1 the ecological mode, to order a quantity $\hat{X}_t$.
- The ordering cost is : $z_t^*(\hat{X}_t) = f_t^{m_1} + f_t^{m_2} + p_t^{m_1}\hat{x}_t^{m_1} + p_t^{m_2}\hat{x}_t^{m_2}$.
- From the decomposition used in the proof of Theorem 1, variables $\hat{x}$ are the optimal basis solution of the following LP:

$$(R_t(\hat{X}_t)) \left\{ \begin{array}{ll} \min & p_t^{m_1} x_t^{m_1} + p_t^{m_2} x_t^{m_2} \\ s.t. & x_t^{m_1} + x_t^{m_2} = \hat{X}_t \\ & (e_t^{m_1} - E_t^{\max}) x_t^{m_1} + (e_t^{m_2} - E_t^{\max}) x_t^{m_2} \leq 0 \\ & x_t^{m_1}, x_t^{m_2} \in \mathbb{R}^+ \end{array} \right.$$

- Key observation: the proportion of products ordered according to each mode does not depend on the quantity $\hat{X}_t$.

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- Let $p_t^{m_1 m_2}$ be the optimal value of $R_t(1)$, and $f_t^{m_1 m_2}$ the quantity $f_t^{m_1} + f_t^{m_2}$: we have $z_t^*(\hat{X}_t) = f_t^{m_1 m_2} + p_t^{m_1 m_2} \hat{X}_t$.

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- We can replace program $IP_t(X_t)$ by the following optimization problem:

$$z_t^*(\hat{X}_t) = \min\{f_t^{uv} + p_t^{uv} \hat{X}_t \mid u, v = 1, \ldots, M \text{ and } (e_t^u - E_t^{\max}) \leq 0\}$$

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  which is the supplying cost of the uncapacitated multi-sourcing lot-sizing problem where, at each period, $O(M^2)$ modes are available.
- The reformulation requires the computation of all costs p_t^{uv}, which can be done in time $O(M^2)$ for each period.

Complexity Analysis

Corollary

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- The algorithmic complexity is increased by a factor M^2.
- Restricted to linear supplying costs.

Solving ULS-PC (1)

Dynamic programming algorithm for the ULS-PC problem

Assumption: $h_t(s_t) = h_t s_t$.

Theorem 3

There always exists an optimal solution $(\widehat{x}, \widehat{s})$ of the ULS-PC problem that satisfies the zero inventory ordering (ZIO) policy (i.e. $\widehat{s}_{t-1}.\sum_{m=1}^{M} \widehat{x}_t^m = 0$ for $t = 1, \ldots, T$).

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Rationale of the algorithm

- Each demand is entirely supplied in a single period,
- At each period t and for each couple of modes m_1 and m_2, a dominant solution $X_t^{m_1} + X_t^{m_2}$ must cover a demand of type $d_{tt'} = \sum_{k=t}^{t'} d_k$,
- At most two modes are used in the same period.

Solving ULS-PC (2)

- Backward dynamic programming algorithm (based on Wagelmans *et al.* 1992).
  - $G(t)$: value of an optimal solution to the instance of ULS-PC with a planning horizon from t to T with $t = 1, \ldots, T$. $G(T+1)$ is equal to zero.
  - $H(t, t')$: best total cost for producing $d_{tt'}$ at period t.

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$$G(t) = \begin{cases} \min\limits_{t < t' \leq T+1} \{G(t') + H(t, t'-1)\}, & \text{if } d_t > 0 \\ \min\left\{G(t+1), \min\limits_{t+1 < t' \leq T+1} \{G(t') + H(t, t'-1)\}\right\}, & \text{if } d_t = 0 \end{cases}$$

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- $G(t)$ can be computed in $O(T)$ if $H(t, t')$ are precomputed.
- $G(1)$ can be computed in $O(T^2)$.
- $H(t, t')$ can be computed $O(TM^2 \log M + T^2)$ (geometric arguments).
- The complexity of the dynamic programming algorithm is $O(TM^2 \log M + T^2)$.

ULS problem with Cumulative Carbon emission constraint ULS-CC

$$\sum_{t'=1}^{t}\sum_{m=1}^{M}(e_t^m - E_{t'}^{\max})x_{t'}^m \leq 0,\ \ t = 1,\ldots,T.$$

Theorem 4

There exists an optimal solution that uses at most two modes in each period: One ecological mode and possibly one non-ecological mode.

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ZIO Property

The cost of the best ZIO policy may be arbitrary large compared to the cost of an optimal policy.

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Parameters	demand	e^u	e^v	$E^{\max}$	p^u	p^v
Period 1	1	0	D+1	D	1	∞
Period 2	2D+1	0	D+1	D	∞	0

- The optimal policy (cost 2): 2 units at period 1 using mode u, $2D$ units at period 2 using mode v.

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- Any finite cost policy must order 2 units using mode u at period 1.

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- The optimal policy (cost 2): 2 units at period 1 using mode u, $2D$ units at period 2 using mode v.
- Any finite cost policy must order 2 units using mode u at period 1.
- The only finite cost ZIO policy consists in ordering all the demands at period 1 using mode u, for a total cost of $2D + 2$.

Complexity analysis of the ULS-CC

Theorem 5

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Proof

- Reduction from a special version of the SubSetSum problem with an additional cardinality constraint on the size of the selected set.

There are n items, each one associated with a weight w_i, together with a knapsack capacity W and an integer k.
The problem consists in selecting at most k objects, allowing multiple copies of items, such that the total weight does not exceed W and is maximized.

The single item ULS problem with Global and Rolling Carbon emission constraints

ULS-GC (Global)

- ULS-GC is *NP*-hard (relaxation of the ULS-CC).

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ULS-RC (Rolling)

- If $R = 1$, then ULS-RC corresponds to the ULS-PC problem, which is polynomial.
- If $R = T$, then ULS-RC is equivalent to the ULS-GC problem which is *NP*-hard.

Lot-sizing problems with periodic carbon emission constraint

Variable and Fixed Carbon Emission Parameter

Fully independent of the supplied quantity

Contributions

ULS-PC with fixed carbon emissions

- ef_t^m: Fixed environmental impact (carbon emissions) related to using mode m at period t.
- ev_t^m: Environmental impact (carbon emissions) related to supplying one unit using mode m at period t.

Periodic carbon emission constraint with fixed carbon emissions

$$\sum_{m=1}^{M}(ev_t^m - E_t^{\max})x_t^m + ef_t^m y_t^m \leq 0, \ t = 1, \ldots, T.$$

ULS-PC with fixed carbon emissions

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Periodic carbon emission constraint with fixed carbon emissions

$$\sum_{m=1}^{M} (ev_t^m - E_t^{\max}) x_t^m + ef_t^m y_t^m \leq 0, \;\; t = 1, \ldots, T.$$

Notation: $\bar{ev}_t^m = ev_t^m - E_t^{\max}$

Observations

- At least one ecological mode $m1$ must be chosen if a quantity is supplied in period t.

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- A minimal threshold order $Q_t^{m_1}$ is required to compensate the fixed carbon emissions of mode $m1$, where $-Q_t^{m_1}\bar{ev}_t^{m_1} = ef_t^{m_1}$.

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- A non-ecological mode cannot be used before fixed carbon emissions are compensated with an ecological mode.

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- A minimal threshold order $Q_t^{m_1 m_2}$ for mode m_1 is required to compensate the fixed carbon emissions of a combination of an ecological mode m_1 and and a non-ecological mode m_2, where $-Q_t^{m_1 m_2}\bar{ev}_t^{m_1} = ef_t^{m_1} + ef_t^{m_2}$.

Observations

- At least one ecological mode $m1$ must be chosen if a quantity is supplied in period t.
- A minimal threshold order $Q_t^{m_1}$ is required to compensate the fixed carbon emissions of mode $m1$, where $-Q_t^{m_1} \bar{ev}_t^{m_1} = ef_t^{m_1}$.
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- Inventory at end of horizon not always zero in optimal solutions. Trivial example: Total demand on horizon smaller than smallest threshold order.

Zero-Inventory-Ordering policy

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  - M=1, T=2.
  - $d_1 = 1$, $d_2 = D$.
  - Cost parameters are null except the holding cost which is unitary.
  - Minimum ordering quantity for single mode is 2.
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- Cost of best ZIO policy may be arbitrary large compared to cost of optimal policy.

Two-mode policy

Theorem 7
There exists an optimal solution for the ULS-PC-F problem that uses at most two modes in each period: One ecological mode and possibly one non-ecological mode.

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Proof: By contradiction

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Theorem 7

There exists an optimal solution for the ULS-PC-F problem that uses at most two modes in each period: One ecological mode and possibly one non-ecological mode.

Proof: By contradiction

- Assume that 3 modes are used in a given period in an optimal solution.
- Different cases + assumptions on carbon emission costs
  - 2 ecological and 1 non-ecological modes
  - 1 ecological and 2 non-ecological modes
  - 3 ecological modes

Complexity analysis

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Complexity analysis

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- General case: The problem is equivalent to a multi-mode lot-sizing problem with a minimal ordering quantity associated to each mode.
- The problem is NP-hard even if only two modes are available and the holding costs are null (reduction from PARTITION).

Conclusion

- The integration of fixed avec variable carbon emission parameters in lot-sizing problems lead to relevant and original problems.
- Complexity analysis.
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- Tax carbon, cap-and-trade, carbon offsets, collaboration SC.