Semidefinite relaxation of the DLSP with sequence-dependent changeover costs

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Plan

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Problem description

Production system

- multiple products
- single-level
- single-resource

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Discrete Lotsizing and Scheduling Problem - DLSP

- Planning horizon divided into short periods
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- Discrete production policy: all-or-nothing assumption
- Constant production capacity

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Complicating feature

Sequence-dependent changeover costs
$\rightarrow$ DLSPSD

Illustrative example

Instance size

2 products, 5 time periods

Instance data

Period	1	2	3	4	5
$p = 1$	1			1	
$p = 2$			1		1

Demand

$p = 1$	10
$p = 2$	20

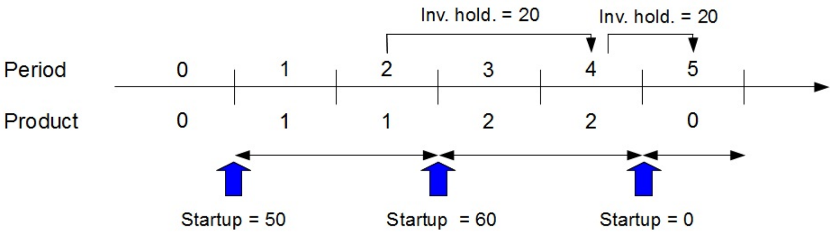
Inv. hold. costs

	0	1	2
0	0	50	75
1	0	0	60
2	0	100	0

Changeover costs

Illustrative example

Optimal production plan



→ Total production cost = 150

QBP formulation

Decision variables

$y_{pt} = 1$ if product p is assigned to period t, 0 otherwise

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Quadratic formulation

$$Z_{DLSP} = min \sum_{p=1}^{P} \sum_{t=1}^{T} h_p \sum_{\tau=1}^{t} (y_{p\tau} - d_{p\tau}) + \sum_{p,q=0}^{P} S_{p,q} \sum_{t=0}^{T-1} y_{pt} y_{qt+1} \quad (1)$$

$$\sum_{\tau=1}^{t} y_{p\tau} \geq \sum_{\tau=1}^{t} d_{p\tau}, \quad \forall p, \forall t \quad (2)$$

$$\sum_{p=0}^{P} y_{pt} = 1, \quad \forall t \quad (3)$$

$$y_{pt} \in \{0, 1\}, \quad \forall p, \forall t \quad (4)$$

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multi-item DLSP: NP-hard problem [Brüggemann and Jahnke 2000]

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Existing solution approaches

- Problem-specific heuristics [Beraldi *et al.* 2008]
- Exact algorithms based on Branch & Bound
  - Key ingredient: quality of the bounds used to evaluate the nodes
  - Bounds obtained by linear reformulation of the QBP

[Belvaux and Wolsey 2001],[Pochet and Wolsey 2006]

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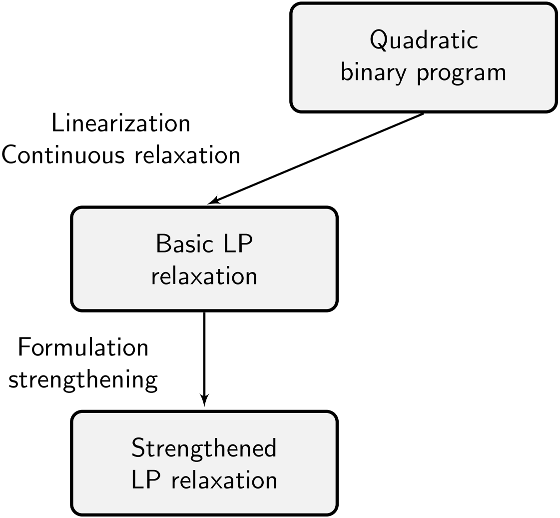
Our proposal

Compute bounds for the DLSPSD thanks to a semidefinite reformulation

State of the art vs Proposed approach



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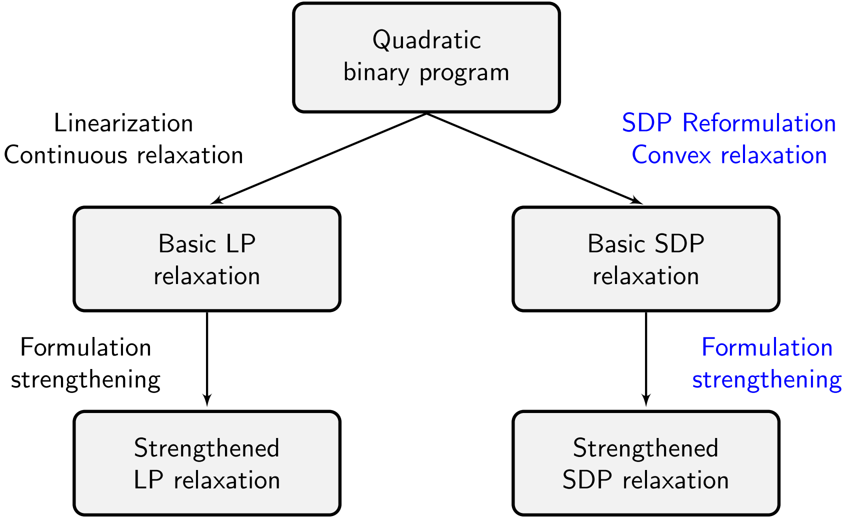

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Semidefinite programming

	Linear programming	Semidefinite programming
Vectorial space	$x \in \mathbb{R}^n$	$X \in S^n$

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Positivity conditions	$x \geq 0 \Leftrightarrow \forall i, x_i \geq 0$	$X \succeq 0 \Leftrightarrow \forall i, \lambda_i \geq 0$

Semidefinite programming

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Optimisation	$\begin{cases} Z_{LP} & = max\ c^T x \\ & a_m^T.x \leq b_m,\ \forall m \\ & x \geq 0 \\ & x \in \mathbb{R}^n \end{cases}$	$\begin{cases} Z_{SDP} & = max < C, X > \\ & < A_m, X > \leq b_m,\ \forall m \\ & X \succeq 0 \\ & X \in S^n \end{cases}$

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Resolution	Simplex algorithm Interior point algorithm	Interior point algorithm Spectral bundle algorithm

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Use in quadratic programming

- Graph problems [Helmberg and Rendl 1998]
- Generic quadratic binary problems
    - Quadratic knapsack problem [Helmberg *et al.* 2000]
    - Quadratic assignment problem [Zhao *et al.* 1998]
- Production management
    - Scheduling [Skutella 1998]
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Solvers

- Primal-dual interior point algorithms: CSDP, DSDP, SeDuMi..
- Spectral bundle methods: SB...

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Quadratic binary program

Detailed formulation

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\begin{cases}
Z_{DLSP} & = min \sum_{p=1}^{P} \sum_{t=1}^{T} h_p \sum_{\tau=1}^{t} (y_{p\tau} - d_{p\tau}) + \sum_{p,q=0}^{P} S_{p,q} \sum_{t=0}^{T-1} y_{pt} y_{qt+1} \\
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Variable redefinition

$x_{pT+t} = 1 - y_{pt} \quad \forall p, \forall t$

i.e. $x_{pT+t} = 1$ if we *do not* produce p in period t, 0 otherwise

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Compact formulation

$$\begin{cases} Z_{DLSP} & = min\ c^T x + x^T \tilde{C} x & \text{Quadratic objective function} \\ & a_{pt}^T x \leq b_{pt} \ \ \forall p, \forall t & \text{Knapsack constraints with pos. coeff.} \\ & e_t^T x = P \ \ \forall t & \text{Equality constraints} \\ & x_i \in \{0,1\}, \ \ \forall i = 1 \ldots n & \text{Binary constraints} \end{cases}$$

SDP reformulation

Introduction of a matrix variable

$$X = \left[\begin{array}{c|c} 1 & x^T \\ \hline x & xx^T \end{array}\right] = \left[\begin{array}{c|cccc} 1 & x_1 & x_2 & \dots & x_n \\ \hline x_1 & x_1^2 & x_1x_2 & \dots & x_1x_n \\ x_1 & x_1x_2 & x_2^2 & \dots & x_1x_2 \\ \vdots & & & & \\ x_n & x_1x_n & x_1x_n & \dots & x_n^2 \end{array}\right] \in S^{n+1}$$

SDP reformulation: objective function

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SDP reformulation: binary constraints

Reformulation of the binary constraints

$x \in \{0, 1\}^n$

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- $x_i \in \{0,1\}, \forall i = 1...n \quad \Leftrightarrow \quad x_i^2 - x_i = 0, \forall i = 1...n$

SDP reformulation: binary constraints

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- $x_i \in \{0,1\}, \forall i = 1...n \quad \Leftrightarrow \quad x_i^2 - x_i = 0, \forall i = 1...n$
- Introduction of matrices $D_i = \left[\begin{array}{c|ccc} 0 & \dots & \text{-0.5} & \dots \\ \hline \vdots & & & \\ \text{-0.5} & & 1 & \\ \vdots & & & \end{array}\right]$

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- Reformulation: $< D_i, X >= 0, \forall i = 1...n$

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Knapsack constraint $a^T x \leq b$

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[Helmberg 2000], [Roupin 2004]

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Square representation of the equality constraints

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$\Leftrightarrow$

Reformulation in S^{n+1}

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\begin{cases}
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Semidefinite relaxation

Convex relaxation

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Initial semidefinite relaxation

$$\begin{cases} Z_{SDP0} & = min \ < C, X > \\ & < A_{pt}, X > \geq 0 \ \ \forall p, \forall t \\ & < E_t, X >= P^2 \ \ \forall t \\ & < D_i, X >= 1 \ \ \forall i = 1...n \\ & < D_0, X >= 0 \\ & X \succeq 0 \end{cases}$$

with $Z_{SDP0} \leq Z_{DSDP}$

Strengthening of the SDP relaxation

Problem-specific valid inequalities

Valid inequalities for the single-product DSLP

[van Eijl and van Hoesel 1997]

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Generic valid inequalities

- Sherali-Adams reformulation of the knapsack constraints
- Binary exclusion between pairs of variables
- Positivity of matrix X coefficients

[Sherali and Adams 1990],[Helmberg 2000],[Roupin 2004]

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SDP reformulation

Quadratic inequalities of the form $x^T \tilde{F} x + f^T x \leq g$

- Introduction of a matrix $F = \left[\begin{array}{c|c} 0 & f^T/2 \\ \hline f/2 & \tilde{F} \end{array}\right]$
- Reformulation: $< F, X > \leq g$

Cutting plane generation

Initial SDP formulation

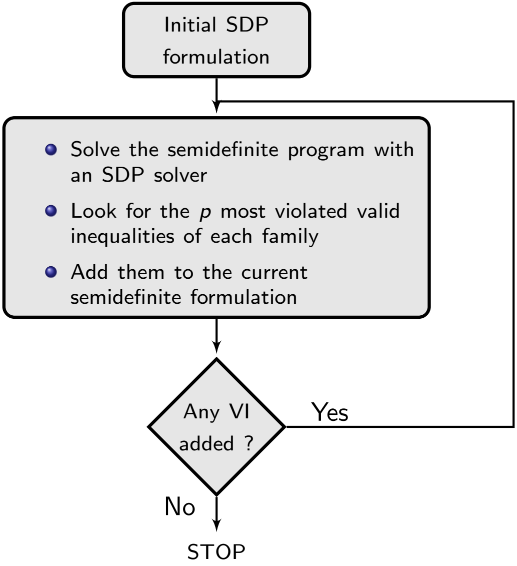
Cutting plane generation

Initial SDP formulation

↓

- Solve the semidefinite program with an SDP solver
- Look for the p most violated valid inequalities of each family
- Add them to the current semidefinite formulation

Cutting plane generation



Plan

Computational experiments

Objective

Comparison between:

- the proposed semidefinite relaxation
- the tigthest linear relaxation previously published for the problem
  - linearization: flow-conservation constraints [Belvaux and Wolsey 2001]
  - shortest-path extended reformulation [Eppen and Martin 1987]

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Method

Computation:

- SDP formulation: DSDP 5.8
- LP/MILP formulation: CPLEX 12.1

Instances

Instance generation

- 100 small instances: 4 to 6 products, 15 to 25 periods
- Capacity utilization: 95%
- Random generation following a procedure described in [Salomon *et al*, 1997]

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Changeover cost structure: two classes of instances

General case

	0	1	2	3	4
0	0	50	75	80	30
1	0	0	60	20	100
2	0	100	0	10	50
3	0	20	70	0	90
4	0	30	60	75	0

Special case: two product families

	0	1	2	3	4
0	0	50	75	80	30
1	0	0	20	50	100
2	0	10	0	80	70
3	0	80	70	0	10
4	0	90	100	25	0

Results: general case

Problem size		Linear relaxation		Semidefinite relaxation	
N	T	Gap_{LP}	$Time_{LP}$	Gap_{SDP}	$Time_{SDP}$
4	15	1.9%	0.1s	0.0%	42s
6	15	0.3%	0.1s	0.0%	86s
4	20	1.3%	0.2s	0.0%	151s
6	20	2.1%	0.2s	0.1%	644s
4	25	1.4%	0.2s	0.1%	713s

Results: product family case

Problem size		Linear relaxation		Semidefinite relaxation	
N	T	Gap_{LP}	$Time_{LP}$	Gap_{SDP}	$Time_{SDP}$
4	15	11.2%	0.1s	0.0%	95s
6	15	4.2%	0.1s	0.0%	145s
4	20	7.2%	0.2s	0.0%	388s
6	20	7.5%	0.2s	0.0%	852s
4	25	7.2%	0.2s	0.2%	1196s

Results: comments

Improved lower bounds

- Average gap decreased:
  - general case: 1.4% $\rightarrow$ 0.04%
  - product family case: 9.5% $\rightarrow$ 0.04%
- Gap fully closed for 97% of the studied instances

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Reformulation as a semidefinite program:

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- Very large extended reformulation in $\mathbb{R}^{(n+1)(n+2)/2}$

Linearization	SDP reformulation
$w_{pqt} = y_{pt} y_{q,t+1}$	$X_{ij} = x_{pt} x_{q,t'}$

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- Inclusion of an infinite number of constraints

$$X \succeq 0 \Leftrightarrow \forall v \in \mathbb{R}^n, \quad v^T X v \geq 0$$

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Very large computation times

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- Computational difficulty of solving a SDP
- Research-based solvers with non-fully optimized BLAS routines
- Resolution of a sequence of SDPs without a warm-start strategy

Plan

Conclusion and perspectives

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- Problem studied: DLSP with sequence-dependent changeover cots
- Main results: very tight lower bounds by semidefinite relaxation
  - Quadratic binary formulation
  - Exploitation of known results for semidefinite relaxation of generic QBP
  - Combination with specific polyhedral results for the DLSP

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- Main results: very tight lower bounds by semidefinite relaxation
  - Quadratic binary formulation
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  - Combination with specific polyhedral results for the DLSP

Perspectives

- Reduce computation times by implementing a warm-start strategy
- Extend the proposed appraoch to other variants of lot-sizing problems

Thank you for your attention !