Algorithms based on time-expanded formulations for Train Timetabling Problems

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November 2022
Introduction to Train Timetabling
Models based on time-expanded graphs
Solution methods
Generalization to skip-stop planning strategies and passenger-centric objectives
Algorithms based on time-expanded formulations for Train Timetabling Problems

Train Timetabling
Railway Optimization Stages

It consists of finding an **optimal schedule** of trains in a railway network satisfying:

- **safety regulations** (e.g., minimum headway times between consecutive trains on the same track) and
- **operational constraints** (e.g., running times, dwell times, station capacity)

The schedule is defined by the **departure and arrival times** of trains at all visited stations.

The **objective function** depends on the railway company (e.g., schedule as many trains as possible).
The railway infrastructure consists of a network with:
- **nodes**: represent the locations where the trains may interact
- **tracks**: connect the nodes and are used by the trains to travel from one node to the next one
The trains to be scheduled are determined based on the passenger demand and can be given in input in two different ways:
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1. A set of train lines (a route between an origin and a destination station with a specific stopping pattern) and a frequency of the train line
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1. A set of train lines (a route between an origin and a destination station with a specific stopping pattern) and a frequency of the train line
2. An ideal timetable for each train provided by the Train Operator that specifies the departure and arrival times at each visited station of the railway network
Constraints

- **minimum headway time** between consecutive trains using the same track
- forbid **overtaking and crossing** of trains on the same track
Constraints

- minimum headway time between consecutive trains using the same track
- forbid overtaking and crossing of trains on the same track
- lower and upper limits on the dwelling time of a train at a station
- lower and upper limits on the running time of a train on a track
Constraints

- **minimum headway time** between consecutive trains using the same track
- forbid **overtaking and crossing** of trains on the same track
- lower and upper limits on the **dwelling time** of a train at a station
- lower and upper limits on the **running time** of a train on a track
- **acceleration and deceleration times** when a train stops at a station
- **maximum number of trains** simultaneously present at a station
- **connection constraints** for passengers transfers
Constraints

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Periodic and non-periodic timetabling

- **Periodic (or cyclic):** the schedule of the trains is repeated every given time period (for example every hour)
Periodic and non-periodic timetabling

- **Periodic (or cyclic)**: the schedule of the trains is repeated every given time period (for example every hour)
- **Non-periodic (or non-cyclic)**: the schedule of the trains is the same every day, it is appropriate for more congested network
Periodic and non-periodic timetabling

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- In this talk, we focus on:
  - Starting from an **ideal timetable for each train**
  - Schedule **as many trains as possible** and **minimize the changes** with respect to the ideal timetables
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- **First the non-periodic problem** (scheduling trains for a day)
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In this talk, we focus on:
- Starting from an ideal timetable for each train
- Schedule as many trains as possible and minimize the changes with respect to the ideal timetables
- First the non-periodic problem (scheduling trains for a day) and then a periodic problem (scheduling trains for one hour)
Models based on time-expanded graphs
Non-periodic Train Timetabling - one-way line

- $S = \{1, \ldots, s\}$: set of stations
- $T$: set of trains each with:
  - an assigned *importance* (e.g., high-speed, local, freight)
  - an *ideal timetable*
Non-periodic Train Timetabling - one-way line

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- $T$: set of trains each with:
  - an assigned importance (e.g., high-speed, local, freight)
  - an ideal timetable
- Time discretization (e.g., one minute)
- The goal is to maximize the total importance of the scheduled trains and minimize the changes to the ideal timetables
Changes to the ideal timetables

Changes can be applied to obtain a feasible timetable (without train conflicts):
Changes to the ideal timetables

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- change the departure and/or arrival times of some trains at some of the visited stations → shift
- increase the dwell time of some trains at some of the visited stations → stretch
- cancel (= not schedule) a train
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- change the departure and/or arrival times of some trains at some of the visited stations → shift
- increase the dwell time of some trains at some of the visited stations → stretch
- cancel (= not schedule) a train

- Lower and Upper limits are imposed for these changes:
  - maximum shift at the departure station for each train
  - maximum total stretch
Algorithms based on time-expanded formulations for Train Timetabling Problems

Models based on time-expanded graphs

Time-expanded graph

- **Time-Space Graph model** by Caprara, Fischetti and Toth (2002)
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  - $V$: train departure $W^i$ and arrival $U^k$ times from/at stations ($i \in S \setminus \{s\}, k \in S \setminus \{1\}$)
Time-expanded graph

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  - $V$: train departure $W^i$ and arrival $U^k$ times from/at stations ($i \in S \setminus \{s\}$, $k \in S \setminus \{1\}$)
  - $A = A_1 \cup \ldots \cup A_{|T|}$: starting, segment (travel), station (stop) and ending arcs
Time-expanded graph

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  - \( A = A^1 \cup \ldots \cup A^{|T|} \): starting, segment (travel), station (stop) and ending arcs
  - \( x_a \): binary variable equal to 1 iff arc \( a \) is selected \((t \in T, a \in A^t)\)
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  - $x_a$: binary variable equal to 1 iff arc $a$ is selected ($t \in T$, $a \in A^t$)

A path in $G$ from $\sigma$ to $\tau$ corresponds to a timetable for a train
### An example

<table>
<thead>
<tr>
<th>Stations</th>
<th>Ideal Timetable A</th>
<th>Ideal Timetable B</th>
<th>Ideal Timetable C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9:00</td>
<td>9:07</td>
<td>9:00</td>
</tr>
<tr>
<td>3</td>
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<td></td>
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<td>4</td>
<td></td>
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<td>10:00</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>9:20</td>
<td>10:20</td>
</tr>
</tbody>
</table>
### An example

**Ideal Timetable A**

<table>
<thead>
<tr>
<th>Stations</th>
<th>Arr. Time</th>
<th>Dep. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9:00</td>
<td>9:07</td>
</tr>
<tr>
<td>2</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
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</tbody>
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**Ideal Timetable B**

<table>
<thead>
<tr>
<th>Stations</th>
<th>Arr. Time</th>
<th>Dep. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9:00</td>
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<td>9:35</td>
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<tr>
<td>3</td>
<td>10:00</td>
<td>10:03</td>
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<tr>
<td>4</td>
<td>10:20</td>
<td>10:24</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Ideal Timetable C**

<table>
<thead>
<tr>
<th>Stations</th>
<th>Arr. Time</th>
<th>Dep. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9:00</td>
<td>9:33</td>
</tr>
<tr>
<td>2</td>
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<td>10:02</td>
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<tr>
<td>3</td>
<td>9:18</td>
<td>10:24</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
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<tr>
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<td></td>
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Algorithms based on time-expanded formulations for Train Timetabling Problems

- Models based on time-expanded graphs
Algorithms based on time-expanded formulations for Train Timetabling Problems

Models based on time-expanded graphs

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ILP arc-model

\[
\max \sum_{t \in T} \sum_{a \in A^t} p_a x_a
\]

- \(p_a\): profit associated with each arc \(a \in A\): importance of the train minus penalties for the changes
ILP arc-model

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\[
\sum_{a \in \delta_t^+(\sigma)} x_a \leq 1, \quad t \in T,
\]

\[
\sum_{a \in \delta_t^-(\nu)} x_a = \sum_{a \in \delta_t^+(\nu)} x_a, \quad t \in T, \nu \in V \setminus \{\sigma, \tau\},
\]
ILP arc-model

\[ \max \sum_{t \in T} \sum_{a \in A^t} p_a x_a \]

- \( p_a \): \textit{profit} associated with each arc \( a \in A \): \textit{importance} of the train minus \textit{penalties} for the changes

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\[ \sum_{a \in \delta^-_t(\nu)} x_a = \sum_{a \in \delta^+_t(\nu)} x_a, \quad t \in T, \nu \in V \setminus \{\sigma, \tau\}, \]

\[ \sum_{a \in C} x_a \leq 1, \quad C \in C, \]

\[ x_a \in \{0, 1\}, \quad a \in A. \]

- \( C \): family of maximal subsets \( C \) of \textit{pairwise incompatible arcs}
Algorithms based on time-expanded formulations for Train Timetabling Problems

- Models based on time-expanded graphs

## ILP path-model

\[
\max \sum_{t \in T} \sum_{p \in P^t} \pi_p x_p
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- \(x_p\): binary variable equal to 1 iff path \(p\) is selected \((t \in T, p \in P^t)\)
- \(\pi_p\): profit associated with each path \(p \in P\): importance of the train minus penalties for the changes along the path
Models based on time-expanded graphs

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\[
\sum_{p \in P^t} x_p \leq 1, \quad t \in T,
\]

\[
\sum_{p \in I} x_p \leq 1, \quad I \in IP,
\]

\(x_p \in \{0, 1\}, \quad P \in P\).

- \(IP\): family of maximal subsets \(I\) of pairwise incompatible paths with incompatibility expressed separately for each station
Solution Methods
Lagrangian-based Heuristic Algorithm

- Proposed in Caprara, Fischetti and Toth (2002)
  extended to a network in Cacchiani, Caprara, Toth (2010)
Lagrangian-based Heuristic Algorithm

- Applied to the ILP arc-model
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- Incompatibility constraints are relaxed in a Lagrangian way
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  - Trains are scheduled one by one, choosing the conflict-free path with maximum Lagrangian profit → Dynamic Programming
Lagrangian-based Heuristic Algorithm

- Applied to the ILP arc-model
- Incompatibility constraints are relaxed in a Lagrangian way
- **Subgradient optimization** to determine near-optimal Lagrangian multipliers
- **Dynamic constraint-generation** is used
- During subgradient optimization, iteratively computes a **heuristic solution**:  
  - Trains are ranked based on the **Lagrangian profit** (original train profit and Lagrangian penalties)
  - Trains are scheduled **one by one**, choosing the conflict-free path with maximum Lagrangian profit → **Dynamic Programming**
  - **Local search** procedures to improve the solution found
Branch-and-Cut-and-Price Algorithm

- Proposed in Cacchiani, Caprara, Toth (2008)
Branch-and-Cut-and-Price Algorithm

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- Solve the LP-relaxation by column generation
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- Pricing problem: determine an optimal path in the time-expanded graph → Dynamic Programming algorithms
- Constraint separation is applied
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- Pricing problem: determine an optimal path in the time-expanded graph → Dynamic Programming algorithms
- Constraint separation is applied
- Branching is applied on the choice of the arcs in the graph
- Constructive heuristics: LP-based fixing of paths or arcs in the graph
Generalization to include additional real-life features
Skip-stop planning strategies
Skip-stop planning strategies\textsuperscript{1}

Skip-stop planning strategies

- An additional change to the ideal timetables: it is possible to skip a stop (= not schedule a stop)

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Skip-stop planning strategies

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- The case study is the high-speed double-track line Beijing-Shanghai in China

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- Two sets of trains:
  - existing trains → actual feasible schedule
  - additional trains → ideal timetables

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- The goal is to increase the capacity utilization of the corridor
- Two sets of trains:
  - existing trains → actual feasible schedule
  - additional trains → ideal timetables
- Acceleration and deceleration times must be taken into account
- maximum number of stops that can be cancelled per train
- no shift for the existing trains

---

Solution method

- ILP arc-model with additional constraints
- Lagrangian-based heuristic algorithm
- Skip-stop strategies (with acceleration and deceleration) are handled by the Dynamic Programming algorithm
Dynamic Programming algorithm

Algorithms based on time-expanded formulations for Train Timetabling Problems

Generalization to include additional real-life features

min travel

maxstr

min travel+acc

maxstr

min travel+dec

maxstr

min travel+acc+dec

maxstr

s

v

u

stop

not stop

stop

not stop

stop

not stop

stop

not stop
Computational experiments - case study

- Beijing-Shanghai corridor: 29 stations
- 304 existing trains and 42 additional trains
Computational experiments - case study

- Beijing-Shanghai corridor: 29 stations
- 304 existing trains and 42 additional trains

- The maximum number of stops that can be cancelled per train is set to 1
- The maximum stretch is set according to the origin-destination of the train
- The maximum shift is set to $\pm 10$, $\pm 20$ or $\pm 30$ minutes
### Computational experiments adding new trains

<table>
<thead>
<tr>
<th>#trains</th>
<th>shift</th>
<th>#sched</th>
<th>travel</th>
<th>stretch</th>
<th>profit</th>
<th>gap%</th>
<th>time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>346 sh±10</td>
<td>109</td>
<td>328(0)</td>
<td>45829</td>
<td>1132(737)</td>
<td>986571</td>
<td>3.72</td>
<td>3857</td>
</tr>
<tr>
<td>346 sh±20</td>
<td>294</td>
<td>333(0)</td>
<td>45827</td>
<td>1142(740)</td>
<td>996286</td>
<td>2.98</td>
<td>6153</td>
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<tr>
<td>346 sh±30</td>
<td>415</td>
<td>336(1)</td>
<td>45681</td>
<td>1161(689)</td>
<td>998975</td>
<td>2.95</td>
<td>9732</td>
</tr>
</tbody>
</table>

Table: No stop skipping
Algorithms based on time-expanded formulations for Train Timetabling Problems

Generalization to include additional real-life features

Computational experiments adding new trains

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<tr>
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Table: With stop skipping
Passenger-centric objectives
Passenger-centric objectives\textsuperscript{2}

- Line Planning Problem $\rightarrow$ frequency of trains for each line in the network

Passenger-centric objectives

- **Line Planning Problem** → **frequency of trains** for each line in the network
- **Regularity (synchronization) constraints** between trains of the same line → to provide a regular service to passengers

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Passenger-centric objectives

- Line Planning Problem $\rightarrow$ frequency of trains for each line in the network
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- Passengers transfer between trains of different lines to reach their destination

---

Passenger-centric objectives\(^2\)

- **Line Planning Problem** $\rightarrow$ *frequency of trains* for each line in the network
- **Regularity (synchronization) constraints** between trains of the same line $\rightarrow$ to provide a regular service to passengers
- Passengers **transfer** between trains of different lines to reach their destination
- Therefore, trains of different lines have to be synchronized effectively

---

Passenger-centric objectives

- Synchronize trains and achieve regularity $\rightarrow$ minimize the total perceived passenger travel time:
Passenger-centric objectives

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Passenger-centric objectives

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Passenger-centric timetabling

- We consider a time period $H$ of one hour (periodic timetabling)
Passenger-centric timetabling

- We consider a time period $H$ of one hour (periodic timetabling).
- Given passengers origin-destination (OD) pairs, we precompute a set of routes for each OD pair $k$ (direct travel options and routes with up to a maximum number of transfer options).
Algorithms based on time-expanded formulations for Train Timetabling Problems

Generalization to include additional real-life features

Passenger-centric timetabling

- We consider a time period $H$ of one hour (periodic timetabling).
- Given passengers origin-destination (OD) pairs, we precompute a set of routes for each OD pair $k$ (direct travel options and routes with up to a maximum number of transfer options).
- $d_k$: number of passengers of OD pair $k$.
- $\pi$: timetabling variables (time of departure and arrival events).

Minimize $\sum_{k \in \text{OD}} d_k \cdot R_k(\pi)$ such that $\pi$ is a feasible timetable, passengers take best routes with respect to $\pi$. $R_k(\pi)$ avg. perceived travel time of one passenger of OD-pair $k$, $\forall k \in \text{OD}$.
Passenger-centric timetabling

- We consider a time period $H$ of one hour (periodic timetabling)
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\text{Min} \sum_{k \in OD} d_k \cdot R_k(\pi)
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Such that $\pi$ is a feasible timetable

- passengers take best routes with respect to $\pi$
- $R_k(\pi)$ avg. perceived travel time of one passenger of OD-pair $k$ \hspace{1cm} $\forall k \in OD$
Average perceived travel time

\[ R_k(\pi) = \frac{1}{d_k} \sum_{v \in V^k} d_k \cdot \frac{L_v^k}{H} \cdot (\gamma_w \cdot W_v^k + Y_v^k) = \frac{1}{H} \sum_{v \in V^k} L_v^k \cdot (\gamma_w \cdot W_v^k + Y_v^k) \]

- \( W_v^k \): adaption time for a route departing in event \( v \) towards the destination of OD-pair \( k \)
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- \( Y_v^k \): in-train time + transfer time on the best route from event \( v \) towards the destination of OD-pair \( k \) (includes penalties for transfers)
- \( V^k \): set of departure events of these routes for OD pair \( k \) (from the origin of \( k \))
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\[ R_k(\pi) = \frac{1}{d_k} \sum_{v \in V_k} d_k \cdot \frac{L^k_v}{H} \cdot (\gamma_w \cdot W^k_v + Y^k_v) = \frac{1}{H} \sum_{v \in V_k} L^k_v \cdot (\gamma_w \cdot W^k_v + Y^k_v) \]

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- Uniformly distributed passenger arrivals in the hour
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- Uniformly distributed passenger arrivals in the hour
- \( L_v^k \): time interval between event \( v \) and the previous departure event of a route for OD-pair \( k \)
- The total number of passengers of OD-pair \( k \) arriving in each interval \( L_v^k \) is \( d_k \cdot \frac{L_v^k}{H} \)
Solution method

- The problem can be modelled as a Periodic Event Scheduling Problem (PESP) (Serafini Ukovich 1989) with additional constraints to compute the $R_k(\pi)$.
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Another approach: PESP without infrastructure constraints + algorithm based on a time-expanded formulation

- Remove from the PESP model all constraints on timetable feasibility → allow conflicts between trains
- Compute passenger-ideal timetables

Lagrangian Heuristic (LH)

Evaluate the impact on passenger perceived travel time → feedback mechanism
Algorithms based on time-expanded formulations for Train Timetabling Problems

Generalization to include additional real-life features

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Lagrangian Heuristic (LH)
Evaluate the impact on passenger perceived travel time $\rightarrow$ feedback mechanism
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  - Evaluate the impact on passenger perceived travel time → feedback mechanism
Feedback mechanism

- After the timetable has been made feasible, some OD-pairs may have a bad perceived travel time
Feedback mechanism

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- We identify the OD-pairs that got the largest worsening
Feedback mechanism

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- We modify the profit structure by penalizing more the shift at origin and intermediate stations where the service was not regular
Feedback mechanism

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- We identify the OD-pairs that got the largest worsening
- We modify the profit structure by penalizing more the shift at origin and intermediate stations where the service was not regular
- Apply again the Lagrangian Heuristic
Algorithms based on time-expanded formulations for Train Timetabling Problems

Generalization to include additional real-life features

Computational experiments - case study

Three case studies of the Dutch railway network (lines of 2019) and one hour period:

- **A2**: 34 stations, 20 trains, 891 OD-pairs.
- **Rotterdam-Groningen**: 77 stations, 60 trains, 3810 OD-pairs.
- **Extended A2**: 140 stations, 88 trains, 11121 OD-pairs.

(a) A2  
(b) Rotterdam-Groningen  
(c) Extended A2
Algorithms based on time-expanded formulations for Train Timetabling Problems

Generalization to include additional real-life features

A2 instance: ideal vs feasible timetable
A2 instance: before and after feedback
Algorithms based on time-expanded formulations for Train Timetabling Problems

Generalization to include additional real-life features

A2 instance: before and after feedback

![Graphs showing data before and after feedback](image-url)
## Comparison

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<tr>
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### Rotterdam Groningen

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Extended A2

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Full PESP

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<td>Ideal + LH + FB</td>
<td>100.10</td>
<td>2 + 0.11</td>
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<td>Full PESP</td>
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<td>- After 2.11 hours</td>
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<td>Lower bound CPLEX</td>
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<td>Rotterdam Groningen</td>
<td>Ideal + LH</td>
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<td>4 + 0.06</td>
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<td>- After 4.18 hours</td>
<td>103.69</td>
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<tr>
<td>Extended A2</td>
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<td>4 + 0.14</td>
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<tr>
<td></td>
<td>Lower bound CPLEX</td>
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Conclusion

- **Time-expanded formulations** can be effectively used in heuristic algorithms for real-life case studies.
- **Efficient timetables** can be computed in planning.

Andrea D'Ariano will talk about efficient methods for train rescheduling during rail operations.
Conclusion

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- Delays and disruptions can still occur in real-time.
Conclusion

- **Time-expanded formulations** can be effectively used in heuristic algorithms for real-life case studies
- Efficient timetables can be computed in planning
- Delays and disruptions can still occur in real-time
- → Andrea D’Ariano will talk about efficient methods for train rescheduling during rail operations
Thank you for your attention