Non-Robust Strong Knapsack Cuts for Capacitated Location-Routing and Related Problems

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Capacitated Location-Routing Problem (CLRP)

Data

- $I$ — set of potential depots with opening costs $f_i$ and capacities $W_i$, $i \in I$
- $J$ — set of customers with demands $d_j$, $j \in J$
- Set of edges $E = E_J \cup E_{IJ}$: $E_J = J \times J$, $E_{IJ} = I \times J$
- $c_e$ — transportation cost of edge $e \in E \cup F$
- An unlimited set of vehicles with capacity $Q$.

The problem

- Decide which depots to open
- Assign every client to an open depot subject to depot capacity
- For every depot, divide assigned clients into routes subject to vehicle capacity
- Minimize the total depot opening and transportation cost
CLRP: an illustration

Figure: LRP instance: $G = (I \cup J, E_J \cup E_{IJ})$
Figure: Location of depots must be jointly decided with vehicle routing.
Literature on the CLRP

- A combination of **two central OR problems**
- \( \approx 3700 \) papers in Google Scholar with both “location” and “routing” in the title

Some recent works on the standard CLRP

- [Belenguer et al. 2011] — important valid inequalities & Branch-and-Cut
- [Baldacci et al. 2011] — exact enumeration & column generation approach
- [Contardo et al. 2014]\(^1\) — state-of-the-art exact algorithm
- [Arnold and Sörensen 2021; Schneider and Löffler 2019] — state-of-the-art heuristics
- [Schneider and Drexl 2017] — the latest survey

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Recently, large improvement in exact solution of classic VRP variants [Pecin et al. 2017] [Pecin et al. 2017] [Pessoa et al. 2018] [Sadykov et al. 2021]

A generic Branch-Cut-and-Price VRP solver [Pessoa et al. 2020]\(^2\) incorporates all recent advances

vrpsolver.math.u-bordeaux.fr

This solver can be applied to the LRP, but problem-specific cuts are necessary for obtaining the state-of-the-art performance

We propose a new family of non-robust cuts, which is shown to be useful for the LRP and some related problems

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Formulation

- $\lambda^i_r, i \in I, r \in R_i$, equals 1 iff route $r$ is used for depot $i$
- $a^r_e, e \in E_J \cup E_{IJ}, r \in \cup_{i \in I} R_i$, equals 1 iff edge $e$ is used by $r$
- $b^r_j, j \in J, r \in \cup_{i \in I} R_i$, equals 1 iff client $i$ is served by route $r$
- $y_i, i \in I$, equals 1 iff route depot $i$ is open
- $z_{ij}, i \in I, j \in J$, equals 1 iff client $j$ is assigned to depot $i$

$$
\min \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{r \in R_i} \sum_{e \in E \cup F} c_e a^r_e \lambda^i_r
$$

$$
\sum_{i \in I} z_{ij} = 1, \quad \forall \ j \in J,
$$

$$
\sum_{r \in R_i} b^r_j \lambda^i_r = z_{ij}, \quad \forall \ i \in I, \ j \in J
$$

$$
\sum_{j \in J} d_j z_{ij} \leq W_i y_i, \quad \forall \ i \in I,
$$

$$
Z_{ij} \leq y_i, \quad \forall \ i \in I, \ j \in J,
$$

$(z, y, \lambda) \in \{0, 1\}^K$
Nested knapsack structure

- $\lambda_r^i, i \in I, r \in R_i$, equals 1 iff route $r$ is used for depot $i$
- $b_r^j, j \in J, r \in \bigcup_{i \in I} R_i$, equals 1 iff client $i$ is served by route $r$
- $z_{ij}, i \in I, j \in J$, equals 1 iff client $j$ is assigned to depot $i$

\[
\sum_{j \in J} d_j z_{ij} \leq W_i, \quad \forall i \in I,
\]

\[
\sum_{r \in R_i} \left( \sum_{j \in J} \sum_{e \in \delta(j)} b_r^j d_j \right) \lambda_r^i \leq W_i, \quad \forall i \in I
\]

- demand
- demand
- demand

- first vehicle load
- second vehicle load
- depot capacity
Master knapsack polytope

\[ \mathcal{P}_{\text{MKP}}(W) = \operatorname{conv} \left\{ t \in \mathbb{Z}_+^W : \sum_{q=1}^W q t_q \leq W \right\}. \]

Theorem ([Aráoz 1974]³)
Each non-trivial facet of \( \mathcal{P}_{\text{MKP}}(W) \) can be described by an inequality of format \( \xi t \leq 1 \) such that \( \xi \in \mathbb{R}_+^W \) is an extreme point of the following system of linear constraints:

\[
\begin{align*}
\xi_1 &= 0, \quad \xi_W = 1, \\
\xi_q + \xi_{W-q} &= 1, \quad \forall 1 \leq q \leq W/2, \\
\xi_q + \xi_q' &\leq \xi_{q+q'}, \quad \forall q + q' \leq W.
\end{align*}
\]

Route Load Knapsack Cuts (RLKC)

\(\theta^i_q \in \mathbb{Z}_+\) — # of routes in \(R_i\) with a total load of exactly \(q\) units.

Theorem

\(\xi^t \leq 1\) defines a non-trivial facet of \(P_{MKP}(W_i)\) if and only if \(\xi^\theta \leq y_i\) defines a non-trivial facet of

\[
\text{conv}\left\{(\theta^i, y_i) \in \mathbb{Z}_+^{W_i} \times \{0, 1\} : \sum_{q=1}^{W_i} q \theta_q \leq W_i y_i\right\}.
\]

Definition

Given a depot \(i \in I\) and a vector \(\xi \in \mathbb{R}_+^{W_i}\) satisfying \(\xi_1 = 0, \xi_{W_i} = 1,\) and \(\xi_q + \xi_{q'} \leq \xi_{q+q'}, \forall q + q' \leq W_i,\) the inequality

\[
\sum_{q=1}^{W_i} \xi_q \theta^i_q \leq y_i
\]

is known as a Route Load Knapsack Cut (RLKC).

Theorem

A Route Load Knapsack Cut is valid for the location-routing formulation.
Separation of RLKCs by Chvátal-Gomory rounding

For any depot \( i \in I \) and any multiplier \( \beta \in \mathbb{R} \) such that \( \beta \geq 1/W_i \), the constraint

\[
\sum_{q=1}^{W_i} \left[ \frac{\beta q}{\beta W_i} \right] \theta_{iq} \leq y_i
\]

is a Route Load Knapsack Cut: superadditivity follows from \( \lfloor r \rfloor + \lfloor r' \rfloor \leq \lfloor r + r' \rfloor \) for all \( r, r' \in \mathbb{R}_+ \).

Separation by enumeration
We consider all multipliers \( \beta = p/q \), such that \( \bar{\theta}_q > 0 \) and \( p = 1, \ldots, q - 1 \).
Separation of \(1/k\)-facets

**Definition**
A master knapsack facet \(\xi x \leq 1\) is called a \(1/k\)-facet if \(k\) is the smallest possible integer such that

\[
\xi q \in \{0/k, 1/k, 2/k, \ldots, k/k\} \cup \{1/2\}.
\]

**Separation by enumeration**
We enumerate all possible \(1/6\)-, \(1/8\)-, and \(1/10\)-inequalities using the algorithm from [Chopra et al. 2015][4].

**Remark**
An \(1/k'\)-inequality is also an \(1/k\)-inequality if \(k'\) is a divisor of \(k\). So, \(1/2\)-, \(1/3\)-, \(1/4\)-, and \(1/5\)-inequalities are also separated.

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Route Load Knapsack Cuts: an example

Data (depot index $i$ is omitted)

$Q = 70$, $W = 140$, $\bar{\theta}_{38} = 1/14$, $\bar{\theta}_{53} = 1/2$, $\bar{\theta}_{65} = 16/14$, $\bar{\theta}_{70} = 1/2$, and $\bar{y} = 277/280$.

Best RLKC obtained by rounding

Multiplier $\beta = 1/53$ gives a RLKC with violation of $\approx 0.082$:

$$\sum_{q=53}^{105} \frac{1}{2} \theta_q + \sum_{q=106}^{140} \theta_q \leq y$$

A better facet-defining $1/6$--inequality

$$\sum_{q=38}^{52} \frac{1}{6} \theta_q + \sum_{q=53}^{87} \frac{3}{6} \theta_q + \sum_{q=88}^{102} \frac{5}{6} \theta_q + \sum_{q=103}^{140} \frac{6}{6} \theta_q \leq y$$

with violation $\approx 0.094$. 
Route Load Knapsack Cuts: an example

1/6–inequality

Ch.-G. rounding inequality
RLKCs and the pricing: label domination

- Pricing problem: Resource Constrained Shortest Path
- It is solved by a labelling algorithm, each label $L$ is $(j^L, \bar{c}^L, q^L)$
- Dominance relation
  \[
  L \succ L' \quad \text{if} \quad j^L = j^{L'}, \quad \bar{c}^L \leq \bar{c}^{L'}, \quad q^L \leq q^{L'}
  \]
- Let $\bar{\mu}(q)$ be the contribution of RLKCs to the reduced cost of a route variable with load $q$
- The same dominance relation is still valid, as $\bar{\mu}(q)$ is non-decreasing:
  \[
  \bar{c}^L \leq \bar{c}^{L'}, \quad q^L \leq q^{L'} \quad \Rightarrow \quad \bar{c}^L + \bar{\mu}(q^L) \leq \bar{c}^{L'} + \bar{\mu}(q^{L'}). 
  \]
RLKCs and the pricing: completion bounds

- A completion bound $B(j, \overline{L})$ is valid if $\overline{c}^L + \bar{\mu}(q^L) + B(j, \overline{L})$ gives a lower bound on the total reduced cost of paths obtained by concatenation of $\overline{L}$ and any $\overline{L} \in \overline{L}$ at node $j$.

- A weak completion bound

$$B_1(j, \overline{L}) = \min_{\overline{L} \in \overline{L}} \left\{ \overline{c}^L \right\}$$

- A tighter completion bound

$$B_2(j, \overline{L}) = \min_{\overline{L} \in \overline{L}} \left\{ \overline{c}^L + \bar{\mu}(q^L) \right\}$$

is valid due to super-additivity of $\bar{\mu}(q)$.

- Completion bounds are used to speed-up labels concatenation and prune labels by bound
Rounded Capacity Cuts (RCC)$^5$

Given a subset of clients $C \subset J$,

$$\sum_{i \in I} \sum_{r \in R_i} \sum_{e \in \delta(C)} a_r^i x_r^i \geq 2 \cdot \left[ \frac{\sum_{i \in C} d_i}{Q} \right].$$

**Separation**

Heuristic algorithms by [Lysgaard et al. 2004]: connected components, max-flow based, greedy construction, local search on previously separated cuts (CVRPSEP reimplemented by us)

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Depot Capacity Cuts (DCC)\(^6\)

If a subset of clients \( C \subset J \) cannot be served by a subset of depots \( S \subset I \),

\[
\sum_{j \in C} d_j > \sum_{i \in S} W_i,
\]

then at least one vehicle from a depot \( i \in I \setminus S \) should visit \( C \):

\[
\sum_{i \in I \setminus S} \sum_{r \in R_i} \sum_{e \in \delta(C)} a_{e}^{r} \lambda_{r}^{i} \geq 2.
\]

Separation

A greedy construction heuristic starting from different seed vertices

COVer inequalities for depots (COV)

Given a subset $J' \subset J$ of customers, such that $\sum_{j \in J'} d_j > W_i$, the following inequality is valid

$$\sum_{j \in J'} z_{ij} \leq (|J'| - 1)y_i.$$  

**Separation**

We solve the MIP for each $i \in I$ such that $\bar{y}_i > 0$

$$\min \sum_{j \in J} (\bar{y}_i - \bar{z}_{ij}) w_j$$

$$\sum_{j \in J} d_j w_j \geq W_i + 1,$$

$$w_j \in \{0, 1\}, \quad \forall j \in J,$$

to check if its solution is less than $\bar{y}_i$. 

Fenchel Cuts over $y$ variables (FC)$^7$

$\hat{Y}$ is the set set of feasible depot configurations

$$
\hat{Y} = \left\{ \hat{y} \in \{0, 1\}^{||l||} : \sum_{i \in I} W_i \hat{y}_i \geq \sum_{j \in J} d_j \right\}.
$$

Separation of $\bar{y} \in \text{conv}(\hat{Y})$

We try to find $\alpha_i \in \mathbb{R}_+^{||l||}$ such that $\sum_{i \in I} \alpha_i \bar{y} < 1$, and $\sum_{i \in I} \alpha_i \hat{y} \geq 1$ for all $\hat{y} \in \hat{Y}$, by solving the LP.

$$
\begin{align*}
\min & \quad \sum_{i \in I} \bar{y}_i \alpha_i \\
\text{subject to} & \quad \sum_{i \in I} \hat{y}_i \alpha_i \geq 1, \quad \forall \hat{y} \in \hat{Y}, \\
& \quad \alpha_i \geq 0, \quad \forall i \in I.
\end{align*}
$$

Chvátal-Gomory Rank-1 Cuts [Jepsen et al. 2008]

Each cut is obtained by a Chvátal-Gomory rounding of a set $C \subseteq J$ of set packing constraints using a vector of multipliers $\rho$ ($0 < \rho_j < 1, j \in C$):

$$\sum_{i \in I} \sum_{r \in R_i} \left\{ \sum_{j \in C} \rho_j \sum_{e \in \delta(j)} \frac{1}{2} a_r^e \right\} \chi_r^i \leq \sum_{j \in C} \rho_j$$

All non-dominated vectors $\rho$ of multipliers for $|C| \leq 5$ are given in [Pecin et al. 2017].

Non-robust in the terminology of [Pessoa et al. 2008]

Separation

Enumeration for $|C| \leq 3$ and a local search heuristic for each non-dominated vector of multipliers for $|C| = \{4, 5\}$. 
Other components of the BCP (through VRPSolver)

▶ Bucket graph-based labelling algorithm for the RCSP pricing [Righini and Salani 2006] [Sadykov et al. 2021]
▶ Partially elementary path ($ng$-path) relaxation [Baldacci et al. 2011]
▶ Automatic dual price smoothing stabilization [Wentges 1997] [Pessoa et al. 2018]
▶ Reduced cost fixing of (bucket) arcs in the pricing problem [Ibaraki and Nakamura 1994] [Irnich et al. 2010] [Sadykov et al. 2021]
▶ Enumeration of elementary routes [Baldacci et al. 2008]
▶ Multi-phase strong branching [Pecin et al. 2017]
  ▶ On number of open depots in a subset of size at most 4 (largest priority)
  ▶ On number of vehicles starting in a depot
  ▶ On the total number of vehicles
  ▶ On number of clients served from a depot
  ▶ On assignment of clients to depots
  ▶ On edges of the graph
Computational results: impact of cuts

“Classic” CLRP test instances by [Prins et al. 2006][8] with 5-10 depot locations and 50-200 clients. Time limit is 12 hours.

BCP₀ — “pure” VRPSolver (without problem-specific cuts)

<table>
<thead>
<tr>
<th>Variant</th>
<th>Root Gap (%)</th>
<th>Time (s)</th>
<th>Nodes</th>
<th>Geomean Time (s)</th>
<th>Solved</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCP₀</td>
<td>4.46%</td>
<td>57.9</td>
<td>19.2</td>
<td>758.7</td>
<td>24/26</td>
</tr>
<tr>
<td>BCPₐₜ−GUB</td>
<td>3.08%</td>
<td>99.0</td>
<td>9.0</td>
<td>481.0</td>
<td>25/26</td>
</tr>
<tr>
<td>BCPₐₜ−DCC</td>
<td>0.85%</td>
<td>101.0</td>
<td>9.1</td>
<td>504.6</td>
<td>24/26</td>
</tr>
<tr>
<td>BCPₐₜ−FC</td>
<td>0.67%</td>
<td>111.4</td>
<td>4.4</td>
<td>283.9</td>
<td>25/26</td>
</tr>
<tr>
<td>BCPₐₜ−RLKC</td>
<td>0.52%</td>
<td>114.7</td>
<td>4.1</td>
<td>264.0</td>
<td>25/26</td>
</tr>
<tr>
<td>BCPₐₜ−COV</td>
<td>0.49%</td>
<td>114.4</td>
<td>4.6</td>
<td>273.4</td>
<td>25/26</td>
</tr>
<tr>
<td>BCPₐₜ</td>
<td>0.48%</td>
<td>115.0</td>
<td>4.1</td>
<td>265.5</td>
<td>25/26</td>
</tr>
</tbody>
</table>

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Computational results: modified instances

\[ \rho = \frac{Q}{\sum_{i \in I} w_i / |I|} \] — “vehicle capacity / depot capacity” ratio

<table>
<thead>
<tr>
<th>Variant</th>
<th>( \rho )</th>
<th>Gap</th>
<th>Time (s)</th>
<th>Nodes</th>
<th>Time (s)</th>
<th>Solved</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCP_0</td>
<td>0.3</td>
<td>2.14%</td>
<td>32.1</td>
<td>18.6</td>
<td>344.7</td>
<td>19/20</td>
</tr>
<tr>
<td>BCP_all−DCC</td>
<td>0.3</td>
<td>0.74%</td>
<td>41.0</td>
<td>12.7</td>
<td>260.9</td>
<td>19/20</td>
</tr>
<tr>
<td>BCP_all−RLKC</td>
<td>0.3</td>
<td>0.51%</td>
<td>48.2</td>
<td>6.1</td>
<td>174.1</td>
<td>19/20</td>
</tr>
<tr>
<td>BCP_all</td>
<td>0.3</td>
<td>0.46%</td>
<td>49.3</td>
<td>5.8</td>
<td>162.9</td>
<td>19/20</td>
</tr>
<tr>
<td>BCP_0</td>
<td>0.5</td>
<td>3.33%</td>
<td>42.9</td>
<td>76.7</td>
<td>2513.6</td>
<td>13/17</td>
</tr>
<tr>
<td>BCP_all−DCC</td>
<td>0.5</td>
<td>2.09%</td>
<td>108.4</td>
<td>35.0</td>
<td>1979.7</td>
<td>13/17</td>
</tr>
<tr>
<td>BCP_all−RLKC</td>
<td>0.5</td>
<td>1.73%</td>
<td>69.8</td>
<td>24.3</td>
<td>1059.3</td>
<td>14/17</td>
</tr>
<tr>
<td>BCP_all</td>
<td>0.5</td>
<td>1.26%</td>
<td>120.3</td>
<td>13.2</td>
<td>813.6</td>
<td>15/17</td>
</tr>
<tr>
<td>BCP_0</td>
<td>0.7</td>
<td>5.94%</td>
<td>51.6</td>
<td>255.3</td>
<td>10511.0</td>
<td>6/17</td>
</tr>
<tr>
<td>BCP_all−DCC</td>
<td>0.7</td>
<td>2.49%</td>
<td>247.7</td>
<td>58.1</td>
<td>4531.6</td>
<td>12/17</td>
</tr>
<tr>
<td>BCP_all−RLKC</td>
<td>0.7</td>
<td>3.91%</td>
<td>83.0</td>
<td>89.4</td>
<td>5438.7</td>
<td>10/17</td>
</tr>
<tr>
<td>BCP_all</td>
<td>0.7</td>
<td>1.53%</td>
<td>284.6</td>
<td>18.9</td>
<td>1734.7</td>
<td>14/17</td>
</tr>
</tbody>
</table>
## Cut generation statistics

### # of generated cuts (# of active cuts at the end of the root)

<table>
<thead>
<tr>
<th>Cut family</th>
<th>Original instances</th>
<th>Modified instances</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RCC</td>
<td>492.5 ( 11.6)</td>
<td>349.5 ( 7.2)</td>
<td>254.9 ( 1.9)</td>
<td></td>
</tr>
<tr>
<td>Im-R1C</td>
<td>7044.0 (215.4)</td>
<td>22405.5 (153.0)</td>
<td>35610.9 (236.2)</td>
<td></td>
</tr>
<tr>
<td>COV</td>
<td>30.8 ( 0.2)</td>
<td>28.1 ( 0.1)</td>
<td>30.7 ( 0.1)</td>
<td></td>
</tr>
<tr>
<td>FC</td>
<td>4.0 ( 0.7)</td>
<td>3.9 ( 0.6)</td>
<td>4.4 ( 0.5)</td>
<td></td>
</tr>
<tr>
<td>GUB</td>
<td>338.5 ( 78.4)</td>
<td>282.4 ( 30.0)</td>
<td>203.2 ( 12.9)</td>
<td></td>
</tr>
<tr>
<td>DCC</td>
<td>488.4 ( 9.9)</td>
<td>1135.1 (10.1)</td>
<td>1636.1 (11.4)</td>
<td></td>
</tr>
<tr>
<td>RLKC (total)</td>
<td>53.4 ( 1.2)</td>
<td>325.1 ( 3.9)</td>
<td>12785.2 (29.6)</td>
<td></td>
</tr>
<tr>
<td>RLKCrround</td>
<td>26.8 ( 0.7)</td>
<td>296.1 ( 3.7)</td>
<td>109.9 ( 0.8)</td>
<td></td>
</tr>
<tr>
<td>RLKC1/2</td>
<td>0.7 ( 0.0)</td>
<td>0.5 ( 0.0)</td>
<td>1401.8 ( 7.7)</td>
<td></td>
</tr>
<tr>
<td>RLKC1/3</td>
<td>0.7 ( 0.0)</td>
<td>0.8 ( 0.0)</td>
<td>675.7 ( 1.6)</td>
<td></td>
</tr>
<tr>
<td>RLKC1/4</td>
<td>1.2 ( 0.0)</td>
<td>1.4 ( 0.1)</td>
<td>418.2 ( 0.9)</td>
<td></td>
</tr>
<tr>
<td>RLKC1/5</td>
<td>1.8 ( 0.0)</td>
<td>1.5 ( 0.0)</td>
<td>1738.4 ( 3.8)</td>
<td></td>
</tr>
<tr>
<td>RLKC1/6</td>
<td>2.6 ( 0.0)</td>
<td>3.2 ( 0.1)</td>
<td>843.4 ( 1.7)</td>
<td></td>
</tr>
<tr>
<td>RLKC1/8</td>
<td>5.7 ( 0.1)</td>
<td>5.0 ( 0.1)</td>
<td>2611.1 ( 4.9)</td>
<td></td>
</tr>
<tr>
<td>RLKC1/10</td>
<td>13.8 ( 0.3)</td>
<td>16.6 ( 0.1)</td>
<td>4986.7 ( 8.2)</td>
<td></td>
</tr>
</tbody>
</table>
### Computational results: comparison with the literature

Time limit is 30 hours

<table>
<thead>
<tr>
<th>Instances</th>
<th>BCP\textsubscript{all}</th>
<th>[Contardo et al. 2014]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Solved</td>
<td>Time</td>
</tr>
<tr>
<td>PPW06</td>
<td>24/26</td>
<td>518</td>
</tr>
<tr>
<td>TB99</td>
<td>9/9</td>
<td>945</td>
</tr>
</tbody>
</table>

| Instances by [Schneider and Löffler 2019]\textsuperscript{9} |
|------------------|---------|----------|---------|----------|
| $|I|$ | $|J|$ | Solved | Improved BKS | Improvement |
| 5  | 100  | 14/14   | 7/14   | 0.05%  |
| 10 | 100  | 14/14   | 5/14   | 0.11%  |
| 10 | 200  | 11/14   | 13/14  | 0.08%  |
| 15 | 200  | 15/20   | 18/20  | 0.12%  |
| 15 | 300  | 6/20    | 11/20  | 0.29%  |
| 20 | 300  | 4/20    | 8/20   | 0.91%  |

VRP-CMD instances

Instances from [Ben Mohamed et al. 2022]¹⁰, occur when solving the 2-echelon stochastic multi-period CLRP.

50 customers, 3-5 already opened depots

VRP with Time Windows and Shifts

Instances with 25-100 customers and 3 shifts

Solved 421 from 504 instances in 30 min. ([Dabia et al. 2019] solved 280)

![Solution Time vs. Number of Solved Instances](image)

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Conclusions

▶ A new family of non-robust strong knapsack cuts for the problems

▶ Exploited monotonicity and superadditivity properties of cuts to limit their impact on the pricing time

▶ These cuts make the BCP algorithm more robust (more harder instances can be solved)

▶ First exact algorithm for the CLRP which can scale to instances with many depot locations

▶ Good results for different problems with the nested knapsack structure.

▶ The paper has been accepted (subject to a minor revision) to the OR journal.


References IV


References V


