Management of the mission of Earth observation satellites Challenge description

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September 25, 2002

Abstract

This document describes in a more formal way a simplified version of the problem of management of the mission of an agile Earth observation satellite, equipped with a high resolution optical instrument, problem informally presented in the previous document.

This simplified problem is the challenge target.

1 Problem simplifications

The problem that is proposed as a challenge results from some simplifications of the problem that has been presented in the previous document. Among them, one can note:

- that all the strips, those dedicated to the complete acquisition of a target as well as those dedicated to the partial acquisition of a polygon, have the same width and the same direction, but not necessarily the same length;
- that there is only two ways of acquiring a strip: in one direction or in the other; let us recall that we call acquisition the association of a strip with an acquisition direction;
- that an earliest and a latest acquisition starting times can be associated with any possible acquisition;

- that the acquisition duration is proportional to the length of the associated strip;
- that an upper bound of the transition time, that is necessary between the end of the first acquisition and the beginning of the second one, can be associated with any pair of possible acquisitions; let us note that, if i and j are two possible acquisitions, the transition time from i to j is not necessarily equal to the transition time from j to i;
- that each strip that is associated with a stereoscopic request must be acquired twice from the same revolution and each time using the same direction (as a consequence, there is only two ways of acquiring a stereo strip, in one direction or in the other); the visibility windows (earliest and latest acquisition starting times) of both acquisitions are moreover *a priori* restricted, in order to met angular acquisition constraints;
- that the downloading constraints, as well as the limitations on the memory and the energy that are available on board, are not taken into account;
- that the uncertainty about the actual acquisition of the selected strips is not taken into account either;
- that the optimization criterion is an additive criterion, which aggregates the gains that are associated with the complete acquisition of the selected targets and with the complete or partial acquisition of the selected polygons, by using a piecewise linear function to assess the gain that results from the partial acquisition of a polygon;
- that the planning horizon (on which strip acquisitions are selected and ordered) is limited to one satellite revolution (in fact one illuminated half revolution).

2 Problem description

The problem to deal with is twofold: one must select a set of strip acquisitions among those that can be performed on the considered horizon and one must order them in time. The expected result is thus a feasible sequence of strip acquisitions. In terms of data, decision variables, constraints, and criterion, the problem can be described as follows:

2.1 Data

The basic data are:

- a number Nr of requests which can be satisfied at least partially from the considered revolution; we assume that they are numbered from 1 to Nr;

- a number Ns of strips which result from the splitting of these requests and can be acquired from the considered revolution; we assume that they are numbered from 1 to Ns; note that two twin strips are associated with each physical strip that results from the splitting of a stereoscopic request, each of them with a specific visibility window;

- a number $Npa = 2.Ns$ of possible strip acquisitions (two for each strip, associated with the two possible acquisition directions); we assume that they are numbered from 1 to Npa, with the following convention: the two possible acquisitions of the strip j, $1 \leq j \leq Ns$, are numbered $2j - 1$ and $2j$;

- for each request i, $1 \leq i \leq Nr$:
  - its type $T[i]$: 0 for a target and 2 for a polygon;
  - its mono or stereo characteristic $St[i]$: 0 for a mono request and 1 for a stereo one;
  - its surface $S[i]$ in square kilometers;
  - the gain $G[i]$ per square kilometer, which can be associated with its complete acquisition;
  - the set $Str[i]$ of the associated strips that can be acquired from the considered revolution;

- for each strip j, $1 \leq j \leq Ns$:
  - the request $R[j]$ the splitting of which it results;
  - its twin strip $Tw[j]$ in case of a stereo request, with the convention that $Tw[j] = 0$ in case of a mono request;
  - its useful surface $Su[j]$ in square kilometers;
  - the duration $Du[j]$ in seconds of the associated acquisition, independent from the acquisition direction;
  - for each of its two ends, referred to as 0 and 1;

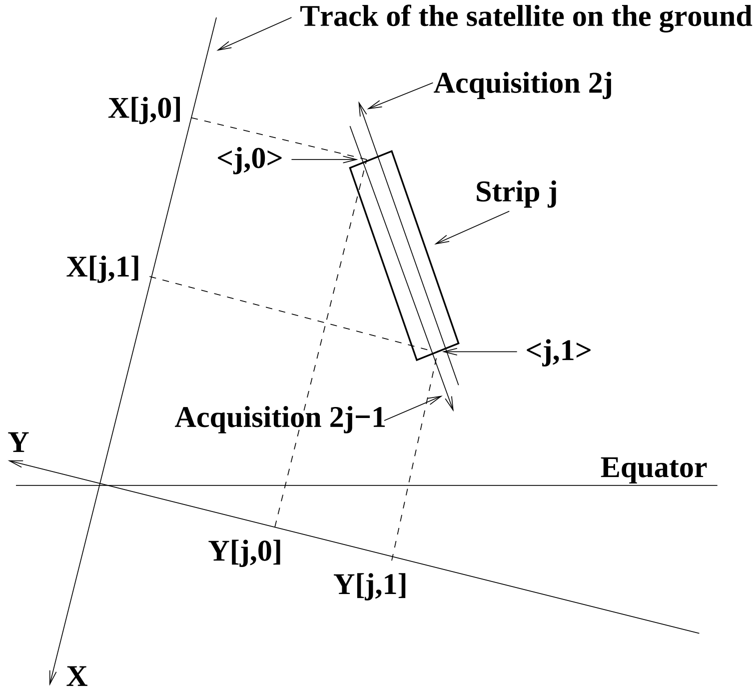

Figure 1: A strip and its two ends.

* its coordinates, in a coordinate system the origin of which is at the intersection between the track of the satellite on the ground with the equator, the x-axis of which is directed towards south, and the y-axis towards west (see Figure 1): $X[j,0]$ and $Y[j,0]$ for the end 0, $X[j,1]$ and $Y[j,1]$ for the end 1;
* its earliest and latest visibility times in seconds: $Te[j,0]$ and $Tl[j,0]$ for the end 0, $Te[j,1]$ and $Tl[j,1]$ for the end 1;

We assume that the acquisition $2j-1$ goes from end 0 to end 1 and that acquisition $2j$ goes on the contrary from end 1 to end 0 (see Figure 1 too).

From these basic data, other data can be computed, such as:

- for each possible acquisition k, $1 \leq k \leq Npa$:
  - its earliest starting time:

$$Tmin[k] = max(Te[j,i],\ Te[j,i'] - Du[j])$$

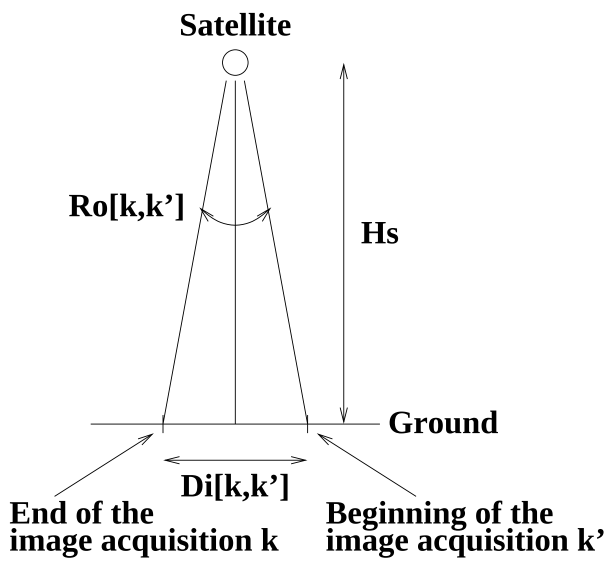

Figure 2: Simplified view of the transition between two strip acquisitions.

– its latest starting time:

$$Tmax[k] = min(Tl[j,i],\ Tl[j,i'] - Du[j])$$

with $j = (k+1)/2$, $i = (k+1) \bmod 2$, and $i' = k \bmod 2$ (the points $< j,i >$ and $< j,i' >$ are respectively the starting and ending points of acquisition k);

- for each pair of possible acquisitions k, k', $1 \leq k, k' \leq Npa$, the distance on the ground in meters between the end of the first one and the beginning of the second one:

$$Di[k,k'] = \sqrt{(X[j,i] - X[j',i'])^2 + (Y[j,i] - Y[j',i'])^2}$$

with $j = (k+1)/2$, $j' = (k'+1)/2$, $i = k \bmod 2$, and $i' = (k'+1) \bmod 2$ (the point $< j,i >$ is the ending point of acquisition k and the point $< j',i' >$ the starting point of acquisition k');

- for each pair of possible acquisitions k, k', $1 \leq k, k' \leq Npa$, an approximation of the rotation in radians that the satellite must perform on itself in order to go from the end of the first one to the beginning of the second one (computed by assuming that it performs this rotation instantaneously and exactly on top of the middle of the segment between the end of the first one and the beginning of the second one):

$$Ro[k,k'] = 2\arctan\frac{Di[k,k']}{2Hs}$$

where Hs is the satellite altitude in meters (problem constant; see Figure 2);

- finally, for each pair of possible acquisitions k, k', $1 \leq k, k' \leq Npa$, an upper bound of the time that is necessary to go from the end of the first one to the beginning of the second one:

  $$Dt[k,k'] = Dmin + \frac{Ro[k,k']}{Vr}$$

  where $Dmin$ is an incompressible transition time in seconds and Vr the maximum speed of rotation on itself of the satellite in radians per second (both problem constants).

2.2 Decision variables

Let s be the selected strip acquisition sequence and let us assume that, with each possible strip acquisition k, $1 \leq k \leq Npa$, one associates:

- a boolean variable $sa[k]$ which equals 1 when acquisition k is selected ($k \in s$) and 0 otherwise;
- a real variable (float) $ta[k]$ which refers to the starting time of acquisition k, in case of selection;

2.3 Constraints

The constraints to met are of four types:

- visibility windows: for each strip acquisition k, $1 \leq k \leq Npa$ that is selected in s:

  $$Tmin[k] \leq ta[k] \leq Tmax[k]$$

- transition times: for each pair of strip acquisitions k, k', $1 \leq k, k' \leq Npa$ that follow each other in s:

  $$ta[k] + Du[k] + Dt[k,k'] \leq ta[k']$$

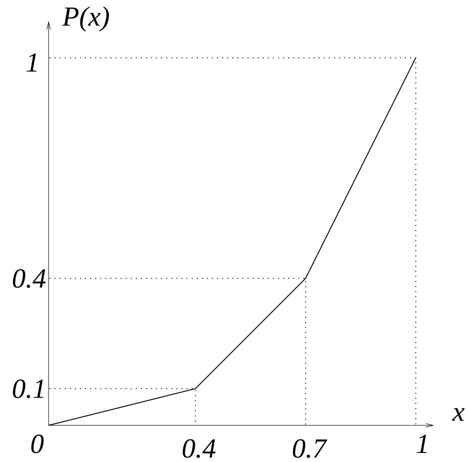

Figure 3: Gain associated with the partial acquisition of a polygon. If $gc[i]$ is the gain that is associated with the complete acquisition of the polygon and $fr[i]$ the percentage of acquired surface, the gain that is associated with $fr[i]$ is $gc[i] \cdot P(fr[i])$

- for each strip j, $1 \leq j \leq Ns$, at most one of the two associated strip acquisitions is selected:

$$sa[2j-1] + sa[2j] \leq 1$$

- for each pair j, j', $1 \leq j, j' \leq Ns$ of twin strips that result from a stereo request (such as $St[R[j]] = St[R[j]] = 1$, $Tw[j] = j'$, $Tw[j'] = j$), either none of them is acquired, or both of them and, in this case, both in the same direction:

$$sa[2j-1] = sa[2j'-1]$$

$$sa[2j] = sa[2j']$$

2.4 Optimization criterion

The criterion to optimize (to maximize) is a gain criterion, which is defined as the sum on all the requests of the gain that is associated with the complete or partial acquisition of each request:

$$g = \sum_{i=1}^{Nr} gr[i]$$

The gain that is associated with the complete acquisition of a request is defined as the product of the surface associated with this request by the gain by surface unit. It is multiplied by 2 in case of a stereo request, in order not to be unfair to these requests that are more consuming in terms of satellite resource:

$$gc[i] = G[i] \cdot S[i] \cdot (St[i] + 1)$$

The gain that is associated with the partial acquisition of a request is defined as the product of the gain that is associated with its complete acquisition by a function of the acquired surface:

$$gr[i] = gc[i] \cdot P(fr[i])$$

The function P is defined on the interval $[0, 1]$. It is piecewise linear and goes through the points $< 0, 0 >$, $< 0.4, 0.1 >$ $< 0.7, 0.4 >$, and $< 1, 1 >$ (see Figure 3). That means for example, that, if 40% of a request is acquired, the associated gain is only 10% of the gain that would be associated with its complete acquisition.

The acquired fraction of a request is defined as the sum of the fractions that are associated with the acquired strips, divided by 2 in case of a stereo request, in order to take into account the fact that every strip of a stereo request must acquired twice:

$$fr[i] = \frac{1}{St[i] + 1} \cdot \frac{1}{S[i]} \cdot \sum_{j \in Str[i]} ss[j] \cdot Su[j]$$

with $ss[j] = sa[2j - 1] + sa[2j]$ (a strip is acquired if and only if one of its associated acquisitions is selected).

Please report to us any inconsistency or incompleteness in this problem description.