One compact and one column generation model for the problem of technicians and interventions in telecom

H. Gavranović

Faculty of Natural Sciences, University of Sarajevo, Zmaja od Bosne 33, 71000 Sarajevo
haris@pmf.unsa.ba

1 Introduction and notations

We present two different models together with their numerical results for the problem that was the subject of Roadef Challenge 2007. The problem seems computationally quite difficult. Even the modest instances are difficult to solve with provable optimum. We abandon from the beginning the idea of optimal method and focus our attention to models and methods that are computationally feasible while providing the solutions close to optimal. Nevertheless, the decomposed model could be further developed to solve medium-size instances with very good solutions, just few percents from the optimal. Both methods are based on the Mathematical Programming approach and most of the ideas come from this theory. The efficient integer programming solver is rather important for the compact model and it is not critical for decomposed model that mainly deals with big size instances. In the eventual quest for the optimality, the importance of efficient solver could be very important.

In order to solve problem, we make two important relaxations common for two models. First, we solve the problem on the day-to-day basis, meaning that we try to group technicians and assign them a batch of interventions to do for only one, current day. For example, on the first day we group the technicians and assign them the most important interventions and then try to solve the next day. The decisions made for one day are never reconsidered, therefore assigned interventions can not be assigned again and do not make a part of the models for the following days. The entire solution consists of the schedule for all working days. The other relaxation deals with interventions. When the number of interventions grows we first try to schedule only the most important, and add the others gradually.

1.1 Notations

When it is possible we will use the the notations from the description of the problem. Other notations will be defined when needed.

2 Compact model

Theoretically, every NP-problem could be modelled with at least on MIP. Compact models are notorious for their beatty, simplicity and they are often natural. They are not always the most efficient. We propose here one such model that solve the problem for only day with available technicians and remaining interventions.

The set $TCH = \{t_1, t_2, \ldots, t_n\}$ represents the set of all technicians available on the given day d. The set $ITV = \{i_1, i_2, \ldots, i_m\}$ represents the set of all interventions not scheduled on the previous days. We introduce the notation of teams representing by the set $TM = \{tm_1, tm_2, \ldots, tm_k\}$ where k is determined heuristically. We have also two ranges $DMN = \{1, 2, \ldots, dmn\}$ for domains and $LVL = \{1, 2, \ldots, lvl\}$ for levels of competencies.

We build a model to assign technicians and interventions to the teams respecting the constraints of competencies and precedences.

The boolean variable x_{t_p,tm_r} has value 1 if the technician t_p make a part of the team tm_r, and zero otherwise. One technician can work in only one team on one day and this fact is represented by the set of constraints :

$$\sum_{tm_r \in TM} x_{t_p,tm_r} = 1, \forall t_p \in TCH$$

The boolean variable y_{i_f,tm_g} has value 1 if the intervention i_f is the responsibility of the team tm_r, and zero otherwise. One intervention can be done, but does not have, by only one team this fact is represented by the set of constraints :

$$\sum_{tm_g \in TM} y_{i_f,tm_g} \leq 1, \forall i_f \in ITV$$

One team can work at most 120 time units by the day so we have the following constraint

$$\sum_{i_p \in ITV} T(y_{i_p}) * y_{i_p,tm_g} \leq 120, \forall tm_g \in TM$$

The constraints that ensure that only qualified teams can do the interventions are more complicated but still natural.

$$y_{i_p,tm_g} * R(y_{i_p}, m, n) \leq \sum_{t_f \in TCH, C(t_f,m) \geq n} x_{t_f,tm_g}, \forall tm_g \in TM, i_p \in ITV, m \in DMN, n \in LVL$$

We still need to ensure that the solution respect the constraints of precedences. The intervention could be done only if all its predecessors are previously done or will be done the same day.

$$\sum_{tm_s \in TM} y_{i_f,tm_s} \leq \sum_{tm_s \in TM} y_{i_g,tm_s}, \forall i_g \in Pred(i_g)$$

This is not enough, we need to be sure that all directed paths in the potential-task graph of interventions are smaller than 120. For the small instances from the set A the number of these constraints is not huge. It was possible to find all paths also for instances B but it does not have to be true in general. In any case, we use this model only on the small instances from set A.

The natural choice for the objective function would be the weighted sum of all interventions. It is possible to use different weights and have competitive results compared to other known methods for the same problem. We propose here, just for the illustration, our final choice for the weights. For the intervention i of priority one, and in a similar way for other priorities, we associate the weight c_i given by the equation

$$c_i = 24 * T(i) * mtn \tag{1}$$

where mtn is the smallest number of technicians needed by the intervention i.

The model, as it is, is not computationally efficient even for very small instances. The main reason is that there is too many symmetrical, equivalent solutions obtained simply by permuting the teams, or interventions, or technicians. On the other hand, it is still useful if we improve it carefully. Different kind of constraints that prevent permutations could be added. For example, it is possible to preassign the technicians to the teams and allowed to a small number of them to change the team [1]. The IP solver was able to solve these models for instances with less than 20 technicians. These results are reported in the table.....

3 Decomposed model

Let me define the batch of interventions as the set of interventions whose total duration does not exceed the 120 time units. We already call the group of technicians team. Let us imagine now that we have all possible combination of teams and batches that are feasible, that is the team have the competencies to effectuate all the interventions from the batch. Let BT represents the set of all feasible combinations of one batch and one team. If we have n technicians and m interventions we can assign to one batch-team one 0-1 column with dimension $n + m$ where every component represents one technician or one intervention. Let a_ij if i correspond to the row representing the technician or the intervention that is a part of batch-team, and 0 otherwise. Let A_j denote the corresponding vector. We then define the matrix A_{BT} of dimension $(n + m) \times |BT|$ with vectors A_j as its columns. We define also the cost $c(A_j)$ associated with the column A_j as the sum of costs given by 2 (this is again only one of all possible choices). One schedule of interventions for one given day is obtained by solving the following set-covering-like problem :

$$\begin{array}{ll} \max \sum_{1\leq j\leq |BT|} x_j c(A_j) & \\ \sum_{1\leq j\leq |BT|} x_j a_{ij} & \leq 1 \ \forall i=1\ldots m+n \\ x_j \text{ is binary.} & \end{array}$$

The solution of this problem need not respect the precedences of interventions. Introducing for $i = n+1\ldots n+m$ new binary variables y_i we add also the constraints to respect the precedences and consequently change the objective function. The new problem has the form

$$\begin{array}{ll} \max \sum_{1\leq j\leq |BT|} x_j c(A_j) - \sum_{n+1\leq i\leq n+m} y_i & \\ \sum_{1\leq j\leq |BT|} x_j a_{ij} & \leq 1 \ \forall i=1\ldots n \\ \sum_{1\leq j\leq |BT|} x_j a_{ij} & \leq y_i \ \forall i=n+1\ldots n+m \\ \text{constraints of precedences on } y_i & \\ x_j, y_i \text{ is binary.} & \end{array}$$

The main difficulty to solve this problem is really huge number of variables x, i.e. huge number of combination batch-team. For some test instances the number of columns is of order 10^{10} and more. Therefore, we solve the problem using the generating columns method. For small and medium-size instances, it is possible, in the given time limit, to solve the linear relaxation of the problem by generation the profitable columns with the help of appropriate IP model. For the bigger instances, a heuristic is need to construct the "good" columns and to solve approximatively the problem 3.

These models are constructed for every day of schedule consecutively. When the interventions are once scheduled they are no more examined. The numerical experimentation shows the strength of the method for some instances.

4 Computational results

In the given table we report the results obtained on the machine with one Pentium IV, 2.4 GHz and with 512 MB of RAM. We use Cplex solver version 9.0 and the solution time was 1400 seconds.

5 Conclusion

It is important to say that would be difficult even to write down the compact model for the whole problem, let alone solving it. Nevertheless, the 3 could be further developed to model whole schedule over several days. We believe, and the tests approve, that it would be interesting and possible to solve this model almost with optimality. Optimal one-day schedule could be constructed in the reasonable time using this model.

The method is parallelizable in several ways. The construction of combinations batch-team could be done in parallel. Every day could be solve for its own in parallel and then try to construct whole solution. Solving set covering problem by branch and bound could be done in parallel.

Références

1. M. Fischetti, A. Lodi, Local Branching, Mathematical Programming B, 98, 23-47, 2003
2. C. Barnhart and E. L. Johnson and G. L. Nemhauser and M. W. P. Savelsbergh and P. H. Vance : Branch-and-price : column generation for solving huge integer programs, Operations Research, 46 , 316–329, 1998
3. M. Desrochers and F. Soumis : A column generation approach to the urban transit crew scheduling problem, Transportation Science, 23, 1–13, 1989
4. E. L. Johnson : Modeling and strong linear program for mixed integer programming, Algorithms and Model Formulations in Mathematical Programming, NATO ASI SEries, 51, 1–41, 1989
5. L. Kroon and M. Fischetti : Crew Scheduling for Netherlands Railways. Destination : Customer. ERIM Report Series Reference No. ERS-2000-56-LIS. Available at SSRN : http ://ssrn.com/abstract=370857, (December 2000)

Tab. 1. Best results for instances set A and B

name	priorities								cost	result
	1		2		3		4			
dataA1	1	75	1	75	1	60	0	0	0	3390
dataA2	2	60	0	0	2	60	0	0	0	5760
dataA3	3	60	3	120	4	60	0	0	0	15120
dataA4	2	120	3	120	5	60	0	0	0	13920
dataA5	6	75	7	120	8	60	0	0	0	34260
dataA6	5	75	5	120	7	30	0	0	0	26940
dataA7	6	120	7	120	8	120	0	0	0	35760
dataA8	4	120	5	60	5	75	0	0	0	23220
dataA9	5	120	7	120	8	120	0	0	0	32400
dataA10	8	120	9	120	10	60	0	0	0	46560
dataB1	5	120	12	75	18	90	25	90	300	47820
dataB2	4	75	5	120	8	120	12	30	300	25770
dataB3	3	105	5	120	9	120	13	60	500	23880
dataB4	7	45	12	15	14	75	17	60	300	48630
dataB5	18	120	26	60	32	120	43	120	890	123840
dataB6	7	120	8	120	13	15	17	120	290	44820
dataB7	6	120	9	120	11	120	18	60	490	42660
dataB8	5	120	8	120	13	120	22	30	490	39030
dataB9	6	120	8	120	8	120	8	120	100	38400
dataB10	7	120	10	120	11	120	11	120	480	46920