A solution approach to the ROADEF/EURO challenge based on Bender's decomposition

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Outline

- Solution approach
- Bender's decomposition (informal)
- MIP model
- Reducing the problem size, and post-optimization
- Results
- Conclusion

Solution approach

Solution approach

- Problem divides naturally into two stages:
  - Stage 1: Select outage dates and reload amounts for each type 2 power plant.
  - Stage 2: Find a production campaign for each scenario independantly.
  - $\Rightarrow$ Bender's decomposition.
- Need a MIP model of the problem. Not straight forward how to model:
  - CT6: Shutdown curve
  - CT12: Maximum modulation
- Strategy: Use solution from Bender's as a guiding solution, and try to "repair" it, i.e., adjusting production levels and reload amounts (will get back to this).

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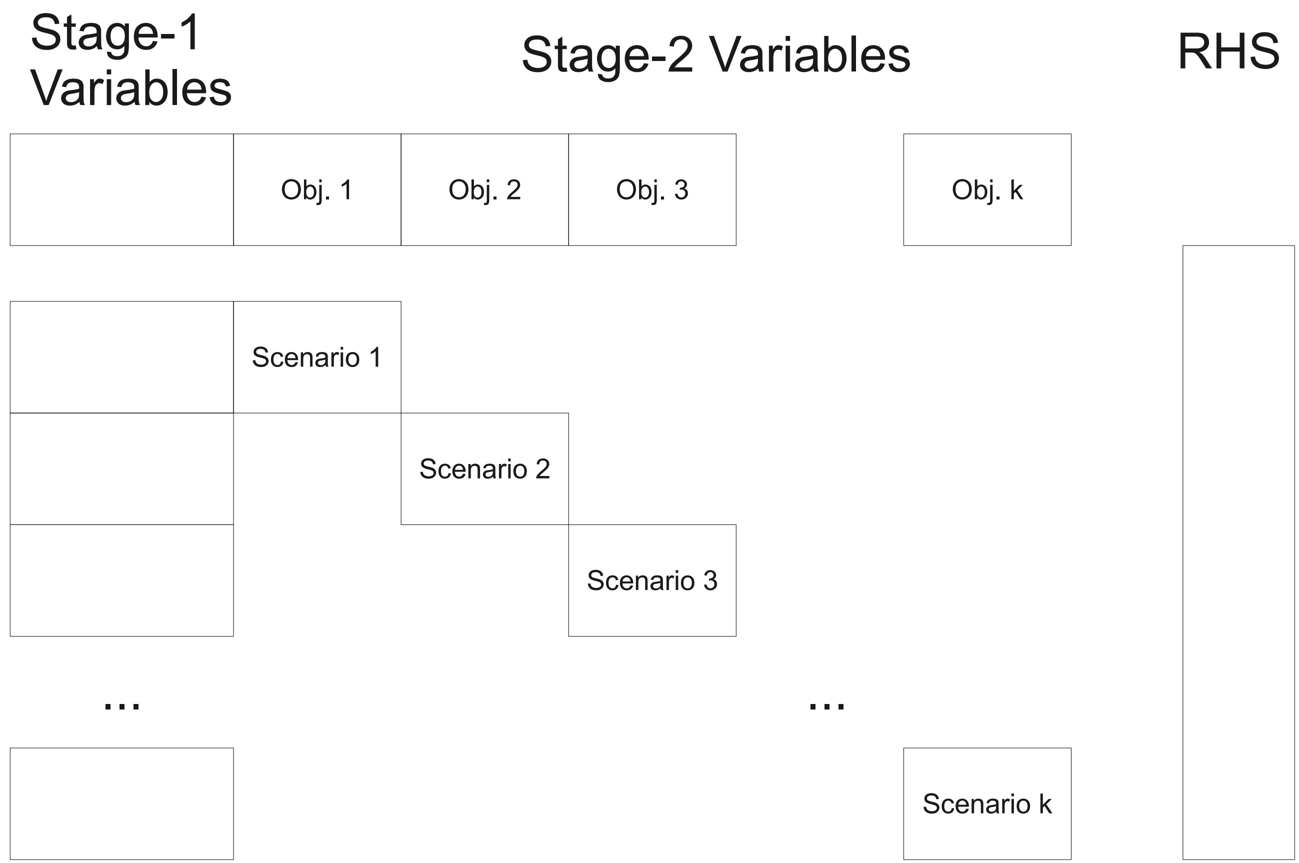
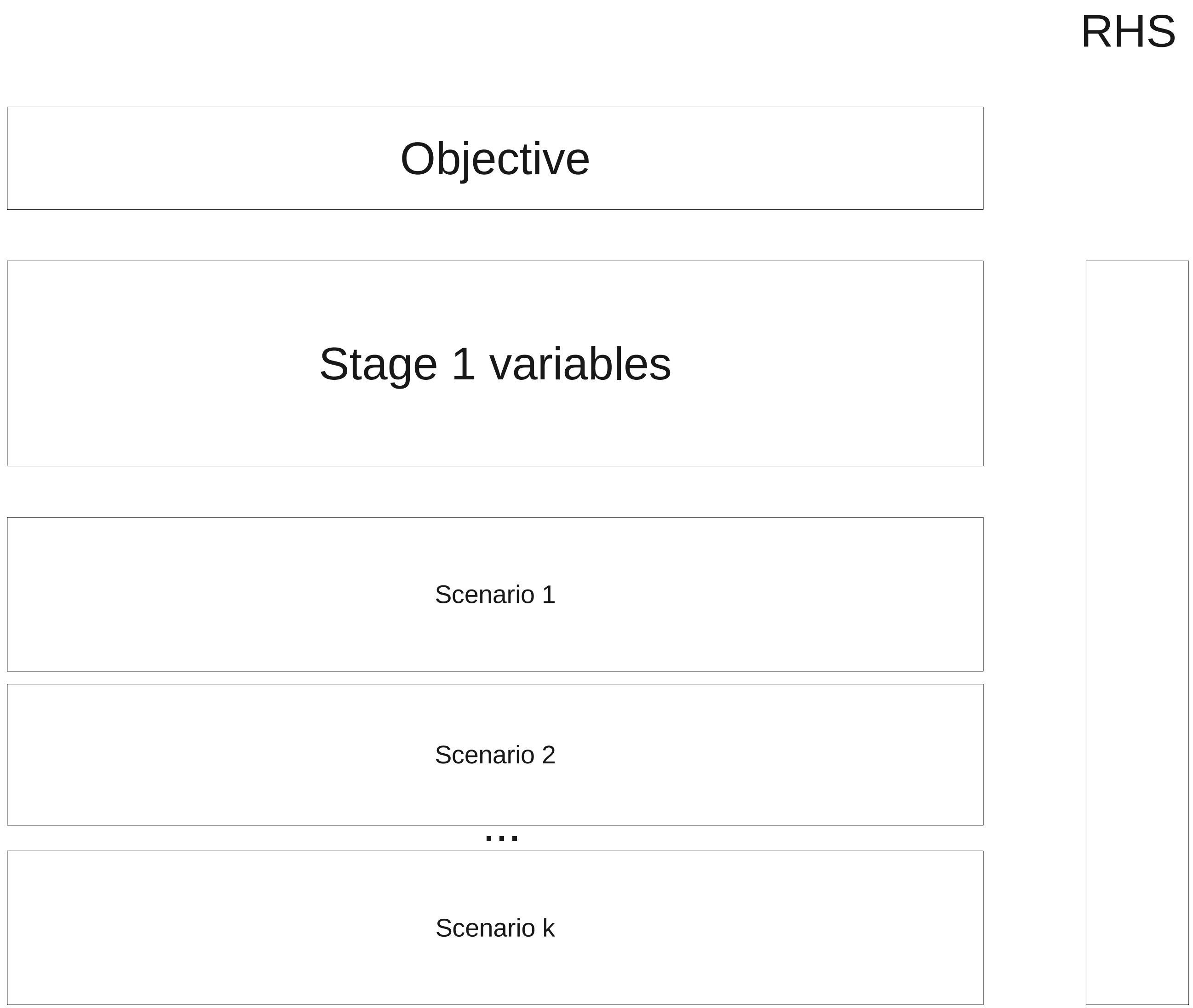
Bender’s Decomposition (informal)

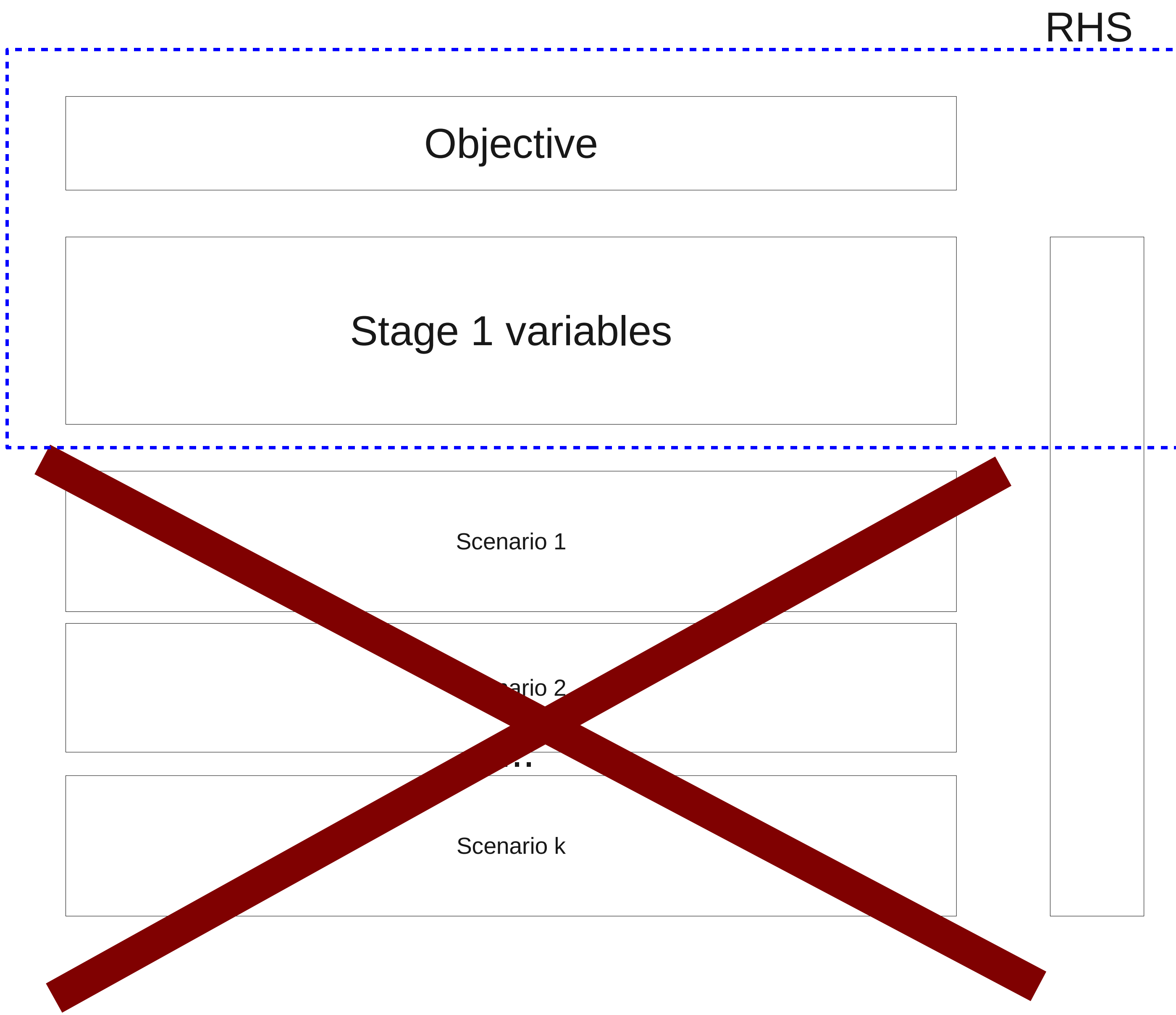
Bender's Decomposition (1)



Bender’s Decomposition (2)



Bender's Decomposition (3)



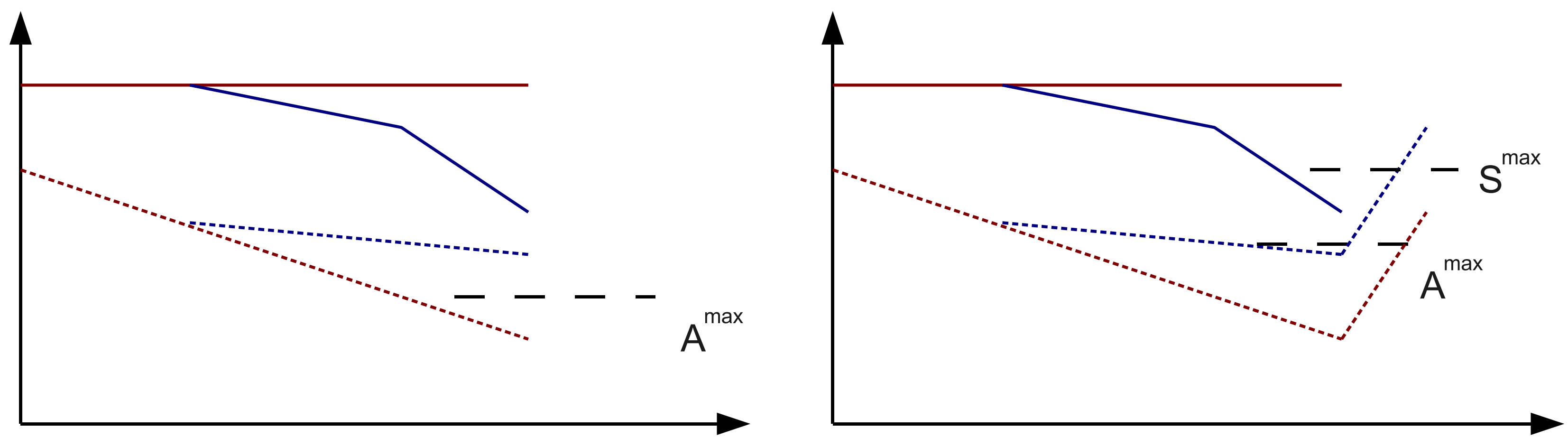
Bender's Decomposition of problem

- Master problem: Select outage dates, and reload amounts for each cycle satisfying:
  - Constraints on cycle outage dates.
  - Constraints on reload amounts.
  - Bender's feasibility/optimality cuts.
  - Additional cuts (explained later).
- Sub-problem (one for each scenario): Given reload amounts, and outage dates for each cycle, find a minimum cost production plan satisfying:
  - Demands.
  - Stock levels below max. before/after reload.
  - (Maximum modulation)
  - (Shutdown curve)

What about shutdown curve, and maximum modulation?

Repair (1)

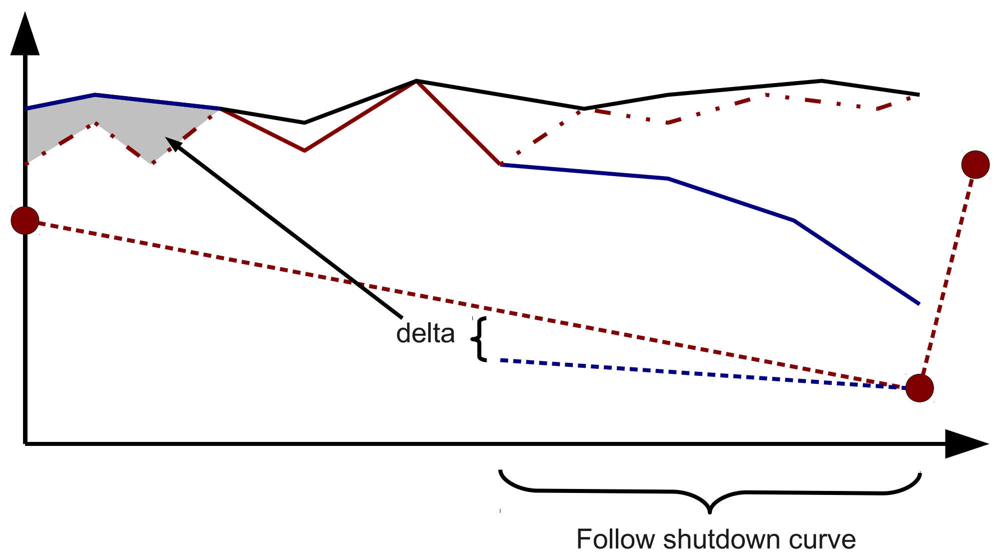
- What are implications of repairing shutdown curve:
  - $\Rightarrow A_{i,k}^{max}$- or $S_{i,k}^{max}$-constraints may become violated.



- What are implications of repairing maximum modulation:
  - $\Rightarrow$ overproduction with regards to demand.

Repair (2)

- Repair-heuristic: Given outage dates/reload amounts, construct a feasible solution “not to far away” from the guiding solution.
- Try to consume excess stock at time t by increasing production levels for times $< t$.



Algorithm outline

Preprocess problem removing infeasible outage dates.
while stop criteria not met **do**
  Solve LP-relaxed Bender's master problem.
  Solve Bender's sub-problems (each scenario).
  if some sub-problem infeasible **then**
    Add Bender's infeasibility cut to master problem.
  else
    Add Bender's optimality cut to master problem.
  end if
end while
while stop criteria not met **do**
  Generate a number of IP-solutions for problem (incl. Bender's cuts).
  Attempt to repair solution.
  Post-optimize solution.
end while
return best found solution.

MIP model

Master problem

$y_{iwk} = 1 \iff$ cycle (i,k) scheduled to start in week w.
r_{ik}: The amount of fuel loaded in cycle (i,k).
W^o_{ik}: Set of allowed outage weeks for cycle (i,k).
$t(w)$: Timestep corresponding to start of week w.

$$\min . \sum_{i\in I}\sum_{k\in K} c_{ik} r_{ik} + 1/|S| \sum_{s\in S} \theta_s$$

$$\text{s.t. } r_{ik} \geq \underline{R}_{ik} \cdot \sum_{w\in W^o_{ik}} y_{iwk} \qquad \forall i \in I, \forall k \in K$$

$$r_{ik} \leq \overline{R}_{ik} \cdot \sum_{w\in W^o_{ik}} y_{iwk} \qquad \forall i \in I, \forall k \in K$$

$$\sum_{w\in W^o_{ik}} y_{iwk} \geq \sum_{w\in W^o_{i,k+1}} y_{i,w,k+1} \qquad \forall i \in I, \forall k \in K$$

$$\sum_{i\in C_m}\sum_{k\in K}\sum_{w\in IT_m}\sum_{w'=w-DA_{ik}+1}^{w} y_{iw'k} \cdot \sum_{t=t(w)}^{t(w+1)-1} P^{max}_{it} \leq I^{max}_m \qquad \forall m \in M_{21}, \forall w \in W$$

$$\sum_{(i,w,k)\in H} y_{iwk} \leq K_H \qquad \forall H \in \mathcal{H}$$

$$r_{ik} \geq 0, y_{iwk} \in \{0,1\}$$

Subproblem

- Subproblem variables (for each scenario):
  - p_{jt}: The production level for type 1 plant j.
  - p_{itk}: production at time t if in cycle (i, k) for type 2 plant i.
  - x^b_{ik}: Fuel at the beginning of cycle (i, k).
  - x^e_{ik}: Fuel at the end of cycle (i, k).
  - x^f_i: Fuel at the end of time horizon for plant i (type 2).
- Subproblem sets and notation:
  - T^p_{ik}: Set of timesteps where cycle (i, k) could be in a production campaign.
  - $K_i(w)$: Set of cycles (i, k) which could be in a production campaign at in week w.
  - $w(t)$: Week containing timestep t.

Subproblem

$$\min . \sum_{t \in T} \sum_{j \in J} c_{jt} \cdot D_t \cdot p_{jt} - \sum_{i \in I} c_{i,T+1} \cdot x_i^f$$

$$s.t.\ x_{ik}^e = x_{ik}^b - \sum_{t \in T} D_t \cdot p_{itk} \qquad \forall (i,k)$$

$$x_{ik}^b = r_{ik} + BO_{ik} \sum_{w \in W_{ik}^o} y_{iwk} + \tilde{Q}_{ik}(x_{i,k-1}^e - BO_{i,k-1} \sum_{w \in W_{ik}^o} y_{iwk}) \qquad \forall (i,k)$$

$$x_{ik}^e \leq A_{i,k+1}^{max} + (1 - \sum_{w \in W_{ik}^o} y_{i,w,k+1})(M_i^1 - A_{i,k+1}^{max}) \qquad \forall (i,k)$$

$$x_{ik}^b \leq S_{i,k}^{max} + (1 - \sum_{w \in W_{ik}^o} y_{iwk})(M_i^1 - S_{ik}^{max}) \qquad \forall (i,k)$$

$$x_i^f \leq \sum_{k' > k} \sum_{w \in W_{ik}^o} y_{iwk'} M_i^1 + x_{ik}^e \qquad \forall (i,k)$$

$$p_{itk} \leq \overline{P}_{it} \cdot \rho(i, w(t), k) \qquad \forall (i, k, t \in T_{ik}^p)$$

$$\underline{P}_{j,t} \leq p_{j,t} \leq \overline{P}_{j,t} \qquad \forall (j,t)$$

$$\sum_{i \in I} \sum_{k \in K_i(w(t))} p_{itk} + \sum_{j \in J} p_{jt} \geq DEM_t \qquad \forall t$$

$$x_{ik}^b \geq 0, x_{ik}^e \geq 0, p_{itk} \geq 0, p_{jt} \geq 0$$

where $M_i^1 = \max_k S_{i,k}^{max}$, $\tilde{Q}_{ik} = \frac{Q_{i,k-1}}{Q_{i,k}}$

Addtional master constraints

- Make subproblems more likely to be feasible:
  - Assume $\overline{P}_{it}$ all points in time to lower bound stock at beginning and end of cycle.
  - Ensure lower bound satisfies A_{ik}^{max}-, and S_{ik}-constraints.
- Make subproblems more likely to be repairable, i.e., closer to satisfying CT6:
  - Use upper bounds on production from the beginning, w of a cycle (i, k) to $w' > w$, assuming an initial fuel level, to bound x_{ik}^b, e_{ik}^e further.
  - This upper bound takes the shutdown curve into account.

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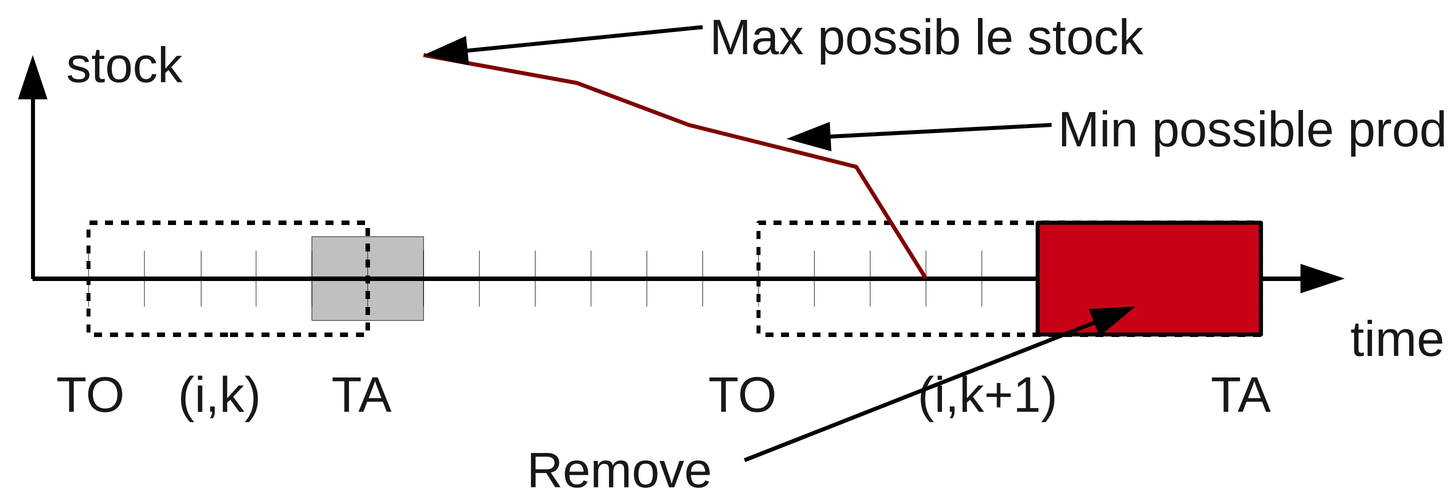
Reducing the problem size, and post-optimization

Preprocessing

- Conflict graph $G = (V, E)$, where
  - every node $v \in V$ corresponds to a possible outage date of a cycle.
  - an edge $(u, v) \in E$, if there is a conflict between outage dates corresponding to u and v.
- Given a subset $S \subseteq V$, such that at least one node in S *must* be picked, any node $u \in V$ incident to all nodes in S can be removed.
- The set of outage dates S for each cycle has this property.
- Very effective.

Heuristic removal of outage dates

- To further reduce the size of the master problem, outage dates may be removed heuristically.
- Assumption: It is not efficient to have a type 2 powerplant with no fuel for too long.
- Given a cycle (i, k), calculate the latest point in time, t_D, where fuel will have been depleated for that cycle.
- Update TA for cycle $(i, k+1)$ such that $TA = (1+\alpha) * t_D$.



Aggregation of subproblems

- To reduce size of scenarios, these are solved on a weekly basis (rather than for every timestep).
- Cutting phase:
  - Not a problem for Bender's cuts, since these are based on variables relating to weeks.
- Solution generating phase (repair):
  - Solution to scenarios must be disaggregated before attempting repair.
  - Done heuristically.
  - If a dissagregation could not be found (one may not exist), then reload amounts are changed, and scenarios are resolved.

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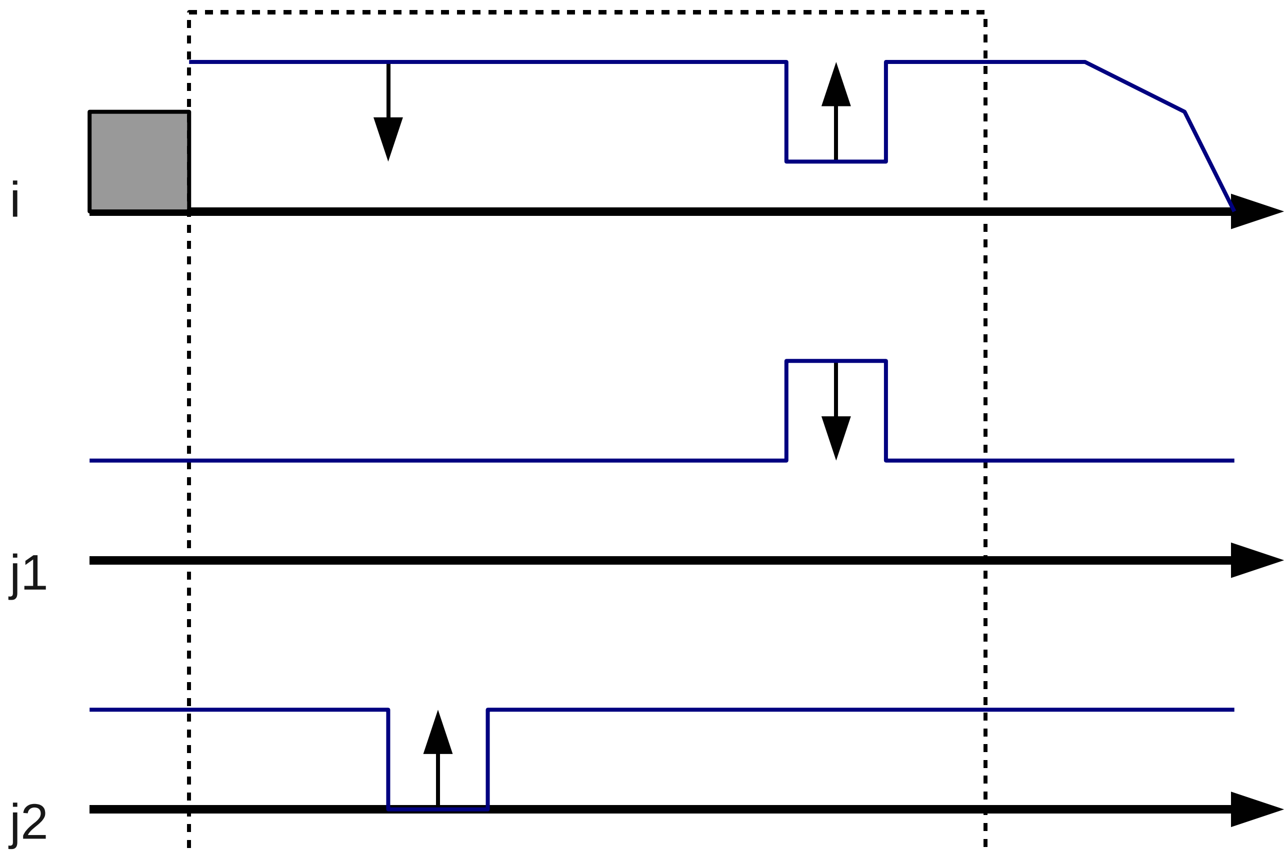
Post-optimization

- Given a solution, shift production between plants, and timesteps to improve cost.
- Do so withinout having to recalculate shutdown curves.



Computational results

Algorithm

Name	#Cuts	#Sols	#Rep.	Dev.	Avg
data0	5	1688	1688	0.0709%	
data1	6	140	129	0.0677%	
data2	39	32	32	0.3119%	
data3	14	35	35	0.2130%	
data4	23	14	14	0.4379%	
data5	20	11	11	0.1818%	0.2139 %

				Value
data6	20	9	9	90.088.820.435
data7	5	10	6	108.315.766.624
data8	1	4	3	2.675.663.995.927
data9	1	1	1	3.660.810.608.977
data10	320	2	2	120.709.135.672

Preprocessing

Name	Total	Rem.	
data0	36	28.78%	
data1	3920	87.42%	
data2	7941	88.39%	
data3	8207	89.74%	
data4	17514	89.37%	
data5	15415	81.91%	
			Rem./TA
data6	24683	85.58%	85.79%
data7	35817	80.61%	80.74%
data8	69481	67.15%	77.87%
data9	69136	62.30%	75.26%
data10	30061	85.43%	85.45 %

Conclusion

- 2-stage approach:
  - Bender’s to add cuts, to restrict the solution space.
  - Generate a number of IP solution, and solve scenarios for each IP solution.
- Ignored CT6, and CT12, and instead repair.
- Aggregate scenarios on a weekly basis.
- Preprocessing removes a big chunk of the problem.
- Gives a lower bound.

Problems:

- Too many binary variables (data8, data9)
  - $\Rightarrow$ master takes long to solve $\Rightarrow$ few cuts/solutions.
- LP-solver + repair-heuristic probably to slow.

Questions & comments

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Thank you!