The Challenge Roadef - Équipe S23

Antoine Rozenknop [1] Roberto Wolfler Calvo [1] Daniel Chemla [2]
Laurent Alfandari [3] Lucas Létocart [1] Guillaume Turri [1]

[1]LIPN, University of Paris Nord, Villetaneuse, France

[2]LMVT, Ecole de Ponts et Chausses, Paris, France

[3]ESSEC, Paris, France

Outline

Problem definitions

- J is the set of M nuclear power plants (type 2).
- I is the set of N thermal power plants (type 1).
- S is the set of possible scenarii.
- T is the set of time steps.

Problem definitions

- J is the set of M nuclear power plants (type 2).
- I is the set of N thermal power plants (type 1).
- S is the set of possible scenarii.
- T is the set of time steps.
- For each thermal $i \in I$, for each scenario $s \in S$ and for each time step $t \in T$ is given:
  - a minimal and a maximal production limits;
  - a production cost $c^s_{i,t}$.

Problem definitions

- J is the set of M nuclear power plants (type 2).
- I is the set of N thermal power plants (type 1).
- S is the set of possible scenarii.
- T is the set of time steps.
- For each thermal $i \in I$, for each scenario $s \in S$ and for each time step $t \in T$ is given:
  - a minimal and a maximal production limits;
  - a production cost $c^s_{i,t}$.
- For each nuclear $j \in J$, is given:
  - a nonnegative cost $r_{j,o}$ represents the fuel cost for each *outage* o.
  - a maximal production limit;
  - a maximal value and a minimal value bounding the quantity of fuel $F_{j,o}$
  - a non linear production profile to respect if fuel is under a threshold BO
  - a set of possible consecutive time steps for stopping for refueling

Problem description

The problem: define a production planning for type 1 and type 2 such that:

- the total demand of energy is satisfied to equality ($d_{t,s}$);
- the number of refueling for each power plant of type 2 is respected (at most five).
- each refueling (*outage*), for each power plant of type 2, should respected a time window.
- each production planning respects the min and max production limits
- the limit on the number of power plants of type 2 stopped together is respected

The problem: define a production planning for type 1 and type 2 such that:

- the total demand of energy is satisfied to equality ($d_{t,s}$);
- the number of refueling for each power plant of type 2 is respected (at most five).
- each refueling (*outage*), for each power plant of type 2, should respected a time window.
- each production planning respects the min and max production limits
- the limit on the number of power plants of type 2 stopped together is respected

Objective: minimize the sum of the production cost for thermal plants plus the refueling cost.

The problem: define a production planning for type 1 and type 2 such that:

- the total demand of energy is satisfied to equality ($d_{t,s}$);
- the number of refueling for each power plant of type 2 is respected (at most five).
- each refueling (*outage*), for each power plant of type 2, should respected a time window.
- each production planning respects the min and max production limits
- the limit on the number of power plants of type 2 stopped together is respected

Objective: minimize the sum of the production cost for thermal plants plus the refueling cost.

Variables (i.e. decisions): $p^s_{j,t}, y^s_{i,t}, e_{j,o}, F_{j,o}, e'_{j,o}$

- $\mathscr{R}$: index set of all production planning for power plants of type 2.
- $\mathscr{R}_j$: production planning for the power plant j of type 2.
- c_ℓ: cost production planning $\ell \in \mathscr{R}$.
- x_ℓ: binary variable equal to 1 iff production planning $\ell \in \mathscr{R}$ is in solution.
- y_i: linear variable representing how much of the demand is covered

$$
\begin{aligned}
(EDF) \quad z(EDF) = \min \sum_{\ell \in \mathscr{R}} c_\ell x_\ell + \sum_{i \in I} \sum_{t \in T} \sum_{s \in S} c^s_{i,t} y^s_{i,t} & \\
s.t. \quad \sum_{\ell \in \mathscr{R}} a^s_{\ell,t} x_\ell + \sum_{i \in I} y^s_{i,t} = d^s_t \quad \forall t \in T, \forall s \in S, & \quad (1) \\
\sum_{\ell \in \mathscr{R}_j} x_\ell = 1 \quad \forall j \in J, & \\
max^s_{i,t} \geq y^s_{i,t} \geq min^s_{i,t} \quad \forall i \in I, \forall t \in T, \forall s \in S, & \\
x_l \in \{0,1\} \quad \forall l \in \mathscr{R}. & \quad (2)
\end{aligned}
$$

The proposed approach can be divided in several Steps briefly described in the following

Step1 Fix the outage and the BO periods

sub-step1 Filtering the data for passing from 5000×500 to 260×1

For each power plant of type 2 create an acyclic graph

sub-step2 Repeat the following three steps until a time limit is elapsed

- For each power plant of type 2 solve a shortest path
- Add the new columns to the master problem and solve it as a LP (i.e. relax constraints 2)
- For each power plant of type 2 update the reduced costs associated to the arcs

sub-step3 Give the Master problem to CPLEX and add all the clique inequalities and solve it to optimality

Step2 Solve the LP problem for fixing the fuel quantity with all scenario, but with weeks instead of time steps

Step3 Solve the LP problem for fixing the production planning with all scenario and time steps

The MIP complexity is given by the number of integer variables, but also by its dimension

- type 1: 100;
- type 2: 58;
- Scenarii: 500;
- Time step: 5000

We aggregate as follows :

- Aggregate time steps into weeks (i.e. from 5000 to 260) noted as h
- Aggregate Scenarii in a single average one (i.e. from 500 to 1) noted as $\bar{s}$

$$
\begin{aligned}
(MIP) \quad z(MIP) = \min & \sum_{l \in \mathscr{R}} c_l x_l + \sum_{i \in I} \sum_{h \in H} c^{\bar{s}}_{i,h} y^{\bar{s}}_{i,h} \\
s.t. \ & \sum_{\ell \in \mathscr{R}} a^{\bar{s}}_{\ell,h} x_\ell + \sum_{i \in I} y^{\bar{s}}_{i,h} = d^{\bar{s}}_h \quad \forall h \in H, \qquad (3) \\
& \sum_{\ell \in \mathscr{R}_j} x_\ell = 1 \quad \forall j \in J, \\
& max^{\bar{s}}_{i,h} \geq y^{\bar{s}}_{i,h} \geq min^{\bar{s}}_{i,h} \quad \forall i \in I, \forall h \in H, \\
& x_l \in \{0,1\} \quad \forall l \in \mathscr{R}.
\end{aligned}
$$

Sub-step 1 – How to build the graph

A column for a power plant of type 2 is a production planning for the time horizon

It can be modeled as a *lot sizing* therefore solved as a shortest path in a graph

(i) In graph G_j, nodes N_j and arcs A_j are constructed once for all at the beginning of the process. Only the (reduced) cost values on the arcs of A_j, that depend on the dual variables output by the linear resolution of the restricted Master Problem, change at every iteration of the Column Generation scheme.

(ii) An arc between two state nodes (h, e) and (h', e') necessarily satisfies the maximum modulation constraint.

(iii) State nodes (h, BO) always exist in all graphs, so that the decreasing profile on the following weeks is the exact profile without using tolerance ϵ.

(iv) The number of discretized levels of stock is equal to a fixed parameter which remains the same for every week of the scheduling horizon.

Sub-step 2 – How calculate the reduced costs

Lets define

$$c_l = \sum_{o \in O} r_{j,o} F_{j,o}$$

But

$$(e_{j,o} - e'_{j,o}) + \sum_{h \in O} a_{j,h} \cong F_{j,o}$$

Therefore

$$c_l = \sum_{o \in O} [(e_{j,o} - e'_{j,o}) r_{j,o} + r_{j,o} \sum_{h \in o} a_{j,h}]$$

Then

$$\bar{c}_l = \sum_{h \in H} (r_{j,h} - u_h) a_{j,h}$$

where u_h is the dual variable associated to constraints (3)

- For each nuclear power plant solve a shortest path on an acyclic graph
- If its reduced cost is negative add the column

It is possible to reinforce the MIP with a set of inequalities, since:

- The number of power plant of type 2 stopped at the same time is limited
- Not overlapping between outage dates is possible

$$
\begin{aligned}
(MIP) \quad z(MIP) = \min & \sum_{l \in \mathscr{R}} c_l x_l + \sum_{i \in I} \sum_{h \in H} c^{\bar{s}}_{i,h} y^{\bar{s}}_{i,h} \\
s.t. & \sum_{\ell \in \mathscr{R}} a^{\bar{s}}_{\ell,h} x_\ell + \sum_{i \in I} y^{\bar{s}}_{i,h} = d^{\bar{s}}_h \quad \forall h \in H, \\
& \sum_{\ell \in \mathscr{R}_j} x_\ell = 1 \quad \forall j \in J, \\
& \sum_{\ell \in \mathscr{R}_c} x_\ell \leq b_c \quad \forall c \in C, \\
& max^{\bar{s}}_{i,h} \geq y^{\bar{s}}_{i,h} \geq min^{\bar{s}}_{i,h} \quad \forall i \in I, \forall h \in H, \\
& x_l \in \{0,1\} \quad \forall l \in \mathscr{R}.
\end{aligned}
$$

$$
\begin{aligned}
(LP1) \quad z(LP1) = \min & \sum_{o \in O} \sum_{j \in J} F_{j,o} r_{j,o} + \sum_{s \in S} \sum_{i \in I} \sum_{h \in H} c^s_{i,h} y^s_{i,h} \\
s.t. \; & \sum_{j \in J} p^s_{j,h} + \sum_{i \in I} y^s_{i,h} = d^s_h \quad \forall h \in H, \forall s \in S \\
& e'_{j,o} + \sum_{h \in o} p^s_{j,h} = e_{j,o} \forall o \in O \\
& e_{j,o} + F_{j,o+1} = e'_{j,o+1} \quad \forall j \in J, \forall o \in O \\
& max^s_{i,h} \geq y^s_{i,h} \geq min^s_{i,h} \quad \forall i \in I, \forall h \in H, \forall s \in S \\
& max^s_{j,h} \geq p^s_{j,h} \geq min^s_{j,h} \quad \forall j \in J, \forall h \in H, \forall s \in S
\end{aligned}
$$

Once fixed the value of refueling and the date of *outage* and *BO* we reintroduce the time step and we fix the rest (i.e. the variable x and y) respecting the decisions already taken

$$
\begin{aligned}
(LP2) \quad z(LP2) = \min & \sum_{s\in S}\sum_{i\in I}\sum_{t\in T} c^s_{i,t} y^s_{i,t} \\
s.t. \; & \sum_{j\in J} p^s_{j,t} + \sum_{i\in I} y^s_{i,t} = d^s_t \quad \forall t \in T, \forall s \in S \\
& e'_{j,o} + \sum_{t\in o} p^s_{j,t} = e_{j,o} \quad \forall o \in O \\
& e_{j,o} + F_{j,o+1} = e'_{j,o+1} \quad \forall j \in J, \forall o \in O \\
& max^s_{i,t} \geq y^s_{i,t} \geq min^s_{i,t} \quad \forall i \in I, \forall t \in T, \forall s \in S \\
& max^s_{j,t} \geq p^s_{j,t} \geq min^s_{j,t} \quad \forall j \in J, \forall t \in T, \forall s \in S
\end{aligned}
$$

- We considered 11 EDF instances partitioned into 2 sets and involving up to 5000 time steps and 250 scenarii.
- Data and best known upper bounds are available at:
  - http://challenge.roadef.org/2010/instances.en.html;
  - http://challenge.roadef.org/2010/qualif.en.html.
- Computing times in seconds of:
  - Squeeze-x64-64 (image based on Debian version sid for AMD64/EM64T) Bi AMD Opteron Dual core 3,2 Ghz
  - 16 GB of physical memory
  - 16 GB of swap memory.

Computational results on Class A instances

	1 thread	1 threads
Name	Objective value	Objective value
data0	$8.73626 * 10^{12}$	$8.73626 * 10^{12}$
data1	$1.75768 * 10^{11}$	$1.70030 * 10^{11}$
data2	$1.53383 * 10^{11}$	$1.46409 * 10^{11}$
data3	$1.61396 * 10^{11}$	$1.54710 * 10^{11}$
data4	$1.22967 * 10^{11}$	$1.12808 * 10^{11}$
data5	$1.38920 * 10^{11}$	$1.28366 * 10^{11}$

Computational results on Class B instances

	1 thread		4 threads	
Name	Objective value	Running time	Objective value	Running time
data6	$8.47103 * 10^{10}$	00:28:50	$8.470769 * 10^{10}$	00:18:00
data7	$8.18527 * 10^{10}$	00:55:20	$8.184168 * 10^{10}$	00:29:55
data8	$8.30129 * 10^{10}$	00:51:11	$8.299788 * 10^{10}$	00:34:09
data9	$8.36119 * 10^{10}$	00:47:45	$8.367746 * 10^{10}$	00:43:25
data10	$7.87675 * 10^{10}$	00:44:27	$7.873125 * 10^{10}$	00:46:52